A Multi-Image Restoration Method for Image Reconstruction from Projections

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Abstract

Traditional Bayesian restoration methods depend heavily on the accuracy of underlying generative models. For the challenging streak noise generated in the procedure of reconstruction from projections, Bayesian methods do not generalize well because accurate signal/noise models are not readily available. In this paper, we reformulate the reconstruction problem into a multi-image based restoration task and demonstrate that multiple images and mutual independence analysis can be utilized to significantly improve the generalization capability of traditional Bayesian frameworks in challenging scenarios. An efficient mutual independence analysis term is designed based on the properties of independent random variables to enforce the independent noise constraint between multiple images in an energy optimization framework, which can effectively detect and correct restoration error due to inaccurate generative models. Quantitative comparisons on phantom image and experiments on clinical scans both show significant improvements in accuracy and robustness of the proposed method.

1. Introduction

Bayesian methods have been extensively used in widely different applications such as image restoration, reconstruction, segmentation and machine learning in the past decades. However, there are still some challenging applications for traditional Bayesian frameworks, especially when accurate prior models are not readily available. For example, the images that are reconstructed from their projections usually suffer streak-type noise due to back projection [8, 14]. The streak noise is signal dependent and looks like line structures. Hence, it is very difficult to model accurately and be discriminated from the true signal. Traditional Bayesian methods are sensitive to inaccurate prior signal/noise models and do not generalize well under such challenging scenarios. In this paper, we mainly address the generalization capability of the Bayesian frameworks in the context of image reconstruction from projections. But other applications could benefit from the proposed ideas as well.

Image reconstruction from projections is a fundamental technique used in Computed Tomography (CT), Optical Coherence Tomography (OCT), Positron Emission Tomography (PET), or Magnetic Resonance Imaging (MRI), among many others. It has been extensively studied in the past [1, 6, 8]. However, the new requirements of the emerging imaging applications pose new challenges to conventional reconstruction methods. For example, faster scanning frame rate in cardiac imaging needs real-time reconstruction algorithm while low-dose CT scanning restricts the number of projections and the signal-to-noise ratio of the projection data. Efficient and effective reconstruction algorithms are critical for such new applications.

Iterative reconstruction scheme provides a way to incorporate accurate signal/noise models. It usually optimizes an objective function that consists of energy terms in both projection domain (e.g., noise models and matching between the measured projections and the forward projection of the reconstructed image) and image domain (e.g., signal models such as smoothness constraint and edge modeling). The photon noise in projection domain can be modeled accurately by Poisson distribution and the signal priors in image domain are extensively studied as well (e.g., [1, 6, 5]). Hence good reconstruction can be achieved. However, the objective function is usually very complex and difficult to optimize due to the complicated transform between image domain and projection domain. Forward/backward projections are necessary in every iteration of the optimization and hence cost significant amount of computation.

A more feasible solution is to integrate signal/noise models with the efficient Filtered Back Projection (FBP) [8, 14] or FFT based reconstruction methods [15]. In [9], an interpolation method in projection domain is proposed to generate more projections and reduce streak noise. Apart from the dramatically increased computation (linearly increased with the number of projections), the prior knowledge dealing with the null space of the projections [16] is usually more difficult than signal/noise modeling in image domain. Postprocessing on the reconstructed image domain can be done based on Bayesian restoration methods such as
[4, 11, 12] to reduce streak noise. However, the streak noise is non-stationary and signal dependent due to the back projection. It is difficult to model the streak noise accurately and discriminate it from the high frequency signal for good noise reduction while preserving the sharp features.

In this paper, we propose a novel method by reformulating the filtered back projection into a multi-image restoration task. This new perspective allows us to address the challenging streak noise and improve the generalization capability of Bayesian frameworks by utilizing multiple images. One largely neglected constraint in traditional multi-image restoration methods is the independence between the noise on different images. It is one of the underlying assumptions in most Bayesian methods for factorizing the joint probability distribution into more tractable terms. However, it is usually neglected and sometimes can be significantly violated in the final results, especially if the generative models are inaccurate. We show that explicitly enforcing the independence analysis between the multiple images can detect and even correct the restoration error caused by inaccurate models in Bayesian frameworks. Hence, a new optimization criteria is designed to regularize the generative models with the cross-group mutual independence analysis. According to the properties of independent random variables, an efficient measurement of mutual independence can be designed, which is evaluated based on summation over a local neighborhood and can be optimized efficiently. This new method, named as Compounded Filtered Back Projection (CFBP), shows significant improvement and robustness to inaccurate models when dealing with the challenging streak noise. It provides better noise reduction while preserving the sharpness and resolution of the signal.

The rest of this paper is organized as follows. Section 2 describes the derivation and implementation of the proposed Compounded Filter Back Projection. Section 3 provides comparisons and evaluation of the new algorithm based on both phantom data and real clinical low-dose CT scans. Conclusions are given in Section 4.

2. Mathematical Formulation

2.1. Filtered Back Projection

A 2D image can be reconstructed from a set of 1D projection measurements. Without loss of generality, we assume the projection measurements, termed sinogram (i.e., $g_f(\rho, \theta)$), are obtained by integrals along parallel rays, where $\theta$ is the angular orientation of the projection rays and the $\rho$ is the distance of the ray to the origin. The projection angle is equally spaced from 0 to $\pi$. The image reconstruction is equivalent to solving the inverse Radon transform based on the given sinogram.

FBP can be derived from the Projection-Slice Theorem. The image is reconstructed by first filtering each 1D projection with a ramp filter and then back projecting all the $P$ filtered projections (i.e., $g_f(\rho, \theta)$) into the 2D image space, as shown in the following equation:

$$\hat{I}(x, y) = \sum_{k=0}^{P-1} g_f(x \cos \theta_k + y \sin \theta_k, \theta_k)$$

However, when the signal-to-noise ratio is low and the number of projections is restricted, FBP method suffers significant streak noise. There are several challenging issues in the FBP procedure:

1. **Amplified noise**: Ramp filter is a high-pass filter and amplifies the high frequency imaging noise in the sinogram dramatically. Low-pass filters can be applied to modulate the ramp filter. However, they inevitably reduce the resolution and blur sharp edges in the image.

2. **Anisotropic streaks**: In the back projection, the noise is projected along the projection direction and looks like line structures. Unlike traditional isotropic noise, streak noise is more difficult to distinguish from sharp edges in the true signal.

3. **Signal-dependency**: High contrast objects, such as bones, cause sharp intensity changes in the sinogram, which are also back projected and cause strong streaks as well. This type of streaks are signal dependent and spatially variant and hence very difficult to build accurate prior models if not impossible.

How to handle the challenging streak noise effectively in Filtered Back Projection has not been carefully addressed in previous literature.

2.2. A new perspective: multiple image restoration

The traditional FBP method is derived based on Projection-Slice Theorem. No priors are assumed about the signal or imaging noise. We can see that, with some rearrangement of the equation (1), the FBP can be interpreted from a new perspective of multiple image restoration. From the new perspective, we can design a more effective streak noise reduction scheme considering various prior knowledge and constraints.

Let $P$ be the total number of projections. It can be divided into $M$ sparsely-sampled subsets of projections. For example, if $M = 2$, the projections are divided into two subsets (even-numbered projections and odd-numbered projections). The two subsets of projections have slightly different projection angles (starting angle is 0 and $\pi/P$ respectively). Let $I_m$ be the FBP reconstruction result from the $m^{th}$ subset of projections (i.e., projections $m, m + M$,...,
where \( m \in [0, M-1] \):

\[
I_m = \sum_{k=0}^{P/M-1} g_f(x \cos \theta_{kM+m} + y \sin \theta_{kM+m}, \theta_{kM+m})
\]

(2)

\( I_m \) provides one version of the reconstruction based on a subset of projections (also referred to as “subreconstruction” in the paper). The traditional FBP then can be reformulated based on the linearity of the Radon transform as follows:

\[
I_{FBP} = \sum_{m=0}^{M-1} I_m
\]

(3)

Since each subset of projections contains slightly angled projections of the same image, each \( I_m \) contains the same underlying true image, but is corrupted by differently oriented streak noise. It can be represented as follows:

\[
I_m = S + N_m \quad m \in [0, M-1]
\]

(4)

where \( S \) is the underlying true image we want to recover and \( N_m \) is the differently oriented streak noise.

Therefore, the reconstruction problem can be reformulated as a multi-image based restoration problem, where we need to estimate the underlying image \( S \) given \( M \) images (i.e., \( I_m \)). Assuming no prior signal/noise models except the independence between the noise terms (i.e., \( N_m \)), the optimal restoration is to average over all images, which is exactly the same as the traditional FBP (up to a normalization constant \( 1/M \)) as shown in equation (3). With prior signal/noise models and a better designed multi-image restoration method, it is possible to provide significant improvements than traditional reconstruction methods. This new perspective provides some important advantages:

1. **Efficiency**: The reconstruction is now based on the sub-reconstructions (i.e., \( I_m \)), which are already in image domain instead of the projection domain. Hence, there is no need of iterative forward and backward projection as in most iterative reconstruction algorithms [1, 6, 5]. The total computation of all sub-reconstructions is exactly the same as the FBP and hence also much more efficient than the idea of interpolation in projection domain [9].

2. **Better utilization of multiple subsets**: Unlike Ordered Subset EM algorithm which down-sample the projections into subsets to speed-up the early stages of iterative reconstruction algorithms [7], we demonstrate that the multiple subsets actually provide more information than one total reconstruction (i.e., FBP). With mutual independence analysis between the multiple sub-reconstructions, we can significantly improve the generalization capability of traditional Bayesian frameworks to deal with inaccurate generative models.

For simplicity, we illustrate our algorithm in this section with two subsets (i.e., \( M = 2 \)). The algorithm can be easily extended to more subsets.

### 2.3. Bayesian methods and independence analysis

Bayesian methods utilize the generative models to estimate the most probable solution. Independence constraint is usually used to factorize the joint probability term into simpler and more tractable terms, but then totally neglected. It is worth more detailed investigation in the multi-image restoration context.

Given multiple sub-reconstructions (e.g., \( I_1 \) and \( I_2 \)), the reconstruction from projections can be formulated as a maximum a posterior (MAP) restoration based on multiple images (i.e., \( \hat{I} = \text{arg max} \ P(S = \hat{I}|I_1, I_2) \)). With the prior signal model (i.e., \( P_S(\cdot) \)) and noise models (i.e., \( P_{N_1}(\cdot) \) and \( P_{N_2}(\cdot) \)) and the assumption that \( N_1 \) and \( N_2 \) are mutually independent, the optimal reconstruction \( \hat{I} \) can be estimated based on the Bayesian rule:

\[
P(\hat{I}|I_1, I_2) = c \cdot P(I_1, I_2|\hat{I})P_S(\hat{I})
\]

\[
= c \cdot P_{N_1}(I_1-\hat{I})P_{N_2}(I_2-\hat{I})P_S(\hat{I})
\]

(5)

where \( c \) is a normalization constant. For simplicity, we only use smooth signal constraint in this section. Edge modeling is further defined in experiment section but it does not affect the derivation anyway. Combined with zero-mean Gaussian noise model, we can define the cost function \( C(\hat{I}) \) of the MAP estimation as follows:

\[
\hat{I} = \text{arg min}_I C(\hat{I}) = \text{arg min}_I (-\log(P(S = \hat{I}|I_1, I_2)))
\]

\[
= \text{arg min}_I (\lambda_1(I_1-\hat{I})^2 + \lambda_2(I_2-\hat{I})^2 + \lambda_3(\hat{I} - \bar{I})^2)
\]

(6)

where \( \bar{I} \) is the average intensity of the true signal in a small neighborhood and \( \lambda_i \) is the weighting of each constraint. The first two terms enforce that the estimated image should look like the observed images (i.e., the sub-reconstructions) because of the zero-mean Gaussian noise model. The 3rd term models the signal properties and prefers smooth signal. More complex models can be used to enforce the smooth signal constraint while preserving the sharp features in the signal (e.g. [2, 11, 12, 13]). The MAP estimation can be obtained when the derivative of \( C(\hat{I}) \) is equal to zero:

\[
\frac{\partial C(\hat{I})}{\partial \hat{I}} = \lambda_1(I_1 - \hat{I}) + \lambda_2(I_2 - \hat{I}) + \lambda_3(\hat{I} - \bar{I}) = 0
\]

(7)

Because the \( \bar{I} \) is unknown, \( C(\hat{I}) \) can be optimized in an iterative manner. At each iteration, \( \hat{I} \) is estimated based on the solution of previous iteration \( \hat{I}^{(k)} \). The iterative minimization process is as follows:

\[
\hat{I}^{(k+1)} = \frac{(\lambda_1 + \lambda_2)\hat{I}_{avg} + \lambda_3\hat{I}^{(k)}}{\lambda_1 + \lambda_2 + \lambda_3}
\]

(8)
where \( I_{avg} = (\lambda_1 I_1 + \lambda_2 I_2) / (\lambda_1 + \lambda_2) \).

The MAP estimation usually achieves good results when the signal/noise models are accurate and the signal and noise have quite different properties [12]. However, when dealing with the challenging streak noise, there are some important issues to address.

First of all, this Bayesian framework is sensitive to the accuracy of the signal/noise models. In equation (6), the weighting factors \( \lambda_i \) should be decided based on the noise variance and the strength of the smoothness constraint. However, the streak noise is non-stationary and difficult to model accurately in advance, which makes it very difficult to set appropriate weighting factors. For example, if the weighting factors on the noise terms (i.e., \( \lambda_1 \) and \( \lambda_2 \)) are set too strong (e.g., \( \lambda_1 = \lambda_2 >> \lambda_3 \)), the MAP estimation becomes \( S = \hat{I} \approx (I_1 + I_2) / 2 \). This solution is clearly not optimal if we have a detailed inspection on the residual noise of the multiple images. The residual noise can be calculated as follows:

\[
\hat{N}_1 = I_1 - \hat{I} \approx I_1 - (I_1 + I_2) / 2 = (I_1 - I_2) / 2 \\
\hat{N}_2 = I_2 - \hat{I} \approx (I_2 - I_1) / 2 = -\hat{N}_1
\]

(9)

We can see that, although the independence between the noise \( N_1 \) and \( N_2 \) is used when deriving the MAP framework, it is severely violated (i.e., \( \hat{N}_1 \approx -\hat{N}_2 \)) when the weighting factors are not set accurately.

Second, the MAP estimation only relies on the average of the multiple images (i.e., \( I_{avg} \)) during the restoration as shown in equation (8). The original multiple images (i.e., \( I_1 \) and \( I_2 \)) are not utilized at all. It is equivalent to applying single image based restoration method on the FBP result, in which case, only the prior generative models can be used to discriminate the signal and noise and hence sensitive to model errors. We believe that the multiple sub-reconstructions actually provide more information than a single reconstruction. Mutual independence analysis between the multiple sub-reconstructions can help discriminate the challenging streak noise from sharp signals and improve the generalization capability of the Bayesian frameworks.

### 2.4. Cross group regularization

The issues in the MAP frameworks are caused by the fact that Bayesian frameworks solely rely on the generative models of the signal and noise without considering the possible restoration errors. It factorizes the joint probability of the noise (i.e., \( P_{N_1,N_2}() \)) into independent terms (i.e., \( P_{N_1}() \cdot P_{N_2}() \)) in equation (5). However, the estimated noise terms (i.e., \( \hat{N}_1 \) and \( \hat{N}_2 \)) might not be independent at all even when there are restoration errors caused by either inaccurate models or converging to local minimum during the optimization. Hence, Bayesian frameworks do not generalize well. It is apparent that explicit mutual independence analysis between the noise on multiple images can provide more information to further detect and correct the possible restoration errors.

For each sub-reconstruction \( I_i \), we can estimate the streak noise \( \hat{N}_i \) by comparing it with the final reconstruction result \( \hat{I} \). Combining with the fact that each sub-reconstruction is the true reconstruction (i.e., \( S \)) corrupted with noise (i.e., \( I_i = S + \hat{N}_i \)), we have

\[
\hat{N}_i = I_i - \hat{I} = N_i + (S - \hat{I})
\]

(10)

If the reconstruction \( \hat{I} \) is perfect (i.e., \( \hat{I} = S \)), the estimated streak noise is equal to the true noise (i.e., \( \hat{N}_i = N_i \)) and should be independent across the sub-reconstructions. However, if there exists reconstruction error (i.e., \( \hat{I} \neq S \)), the error (i.e., \( S - \hat{I} \)) will be equally reflected in all estimated streak noises as shown in equation (10) and will cause significant dependence. Therefore, analyzing and minimizing the dependence between the estimated streak noises can help identify and correct reconstruction errors. Mutual information is a common way to measure dependence:

\[
MI(\hat{N}_1, \hat{N}_2) = \sum_{\hat{N}_1} \sum_{\hat{N}_2} p(\hat{N}_1, \hat{N}_2) \log \frac{p(\hat{N}_1, \hat{N}_2)}{p(\hat{N}_1)p(\hat{N}_2)}
\]

(11)

To explicitly enforce the independent streak noise constraint, we propose to incorporate mutual independence into the traditional MAP framework as an additional energy term. The new reconstruction algorithm can then be described as the following optimization problem:

\[
\hat{I} = \arg \min_{\hat{I}} C(\hat{I}) = \arg \min_{\hat{I}} (\lambda_0 (\hat{I} - I)^2 + \lambda_1 (I_1 - \hat{I})^2 + \lambda_2 (I_2 - \hat{I})^2 + \lambda_3 MI(\hat{N}_1, \hat{N}_2))
\]

(12)

As we can see, if the Bayesian estimation has accurate models and provides a good reconstruction result, the estimated noise on different images is also accurate and satisfies the independence constraint. Therefore, the new mutual independence term will penalize the solution and make it less likely to happen. With the additional independence analysis, the new algorithm not only relies on the \( I_{avg} \) but also fully utilizes the multiple sub-reconstructions for better noise reduction.

### 2.5. Compounded Filtered Back Projection

The new proposed reconstruction method is named as Compounded Filtered Back Projection (CFBP) and consists of two stages. First, the projections are divided into several subsets and one sub-reconstruction is obtained from each subset of projections by FBP. At the second stage, an optimization procedure as shown in equation (12) is carried on
in image domain to find the optimal reconstruction that satisfies both the generative models defined in subsection 2.3 and the cross-group independence analysis term between the multiple sub-reconstructions defined in subsection 2.4.

Since the computation of FBP increases linearly with the number of projections, the total computational cost in the first stage of the CFBP is the same as doing FBP on all projections. Iterative optimization procedure might be needed for the second stage. But it is much faster than traditional iterative reconstruction methods because no forward and backward projections are necessary during the iterations.

Strict evaluation and optimization of the mutual information could be very expensive [10, 3]. Instead we propose an efficient approximation based on one of the basic properties of the independent random variables:

\[
E(h_1(\hat{N}_1)h_2(\hat{N}_2)) = E(h_1(\hat{N}_1))E(h_2(\hat{N}_2))
\]  

(13)

where \( E \) is the expectation, and \( h_1() \) and \( h_2() \) are functions of \( \hat{N}_1 \) and \( \hat{N}_2 \) respectively. This equation holds for arbitrary \( h_1() \) and \( h_2() \), as long as the random variables \( \hat{N}_1 \) and \( \hat{N}_2 \) are independent.

For efficient computation, we choose the functions to be a weighted sum of different moments. For example, if we choose \( h_3(\hat{N}_1) = \hat{N}_1 \) and \( h_2(\hat{N}_2) = \hat{N}_2 \), it reduces the independent constraint to the uncorrelation constraint. When the random variables follow Gaussian distribution, independence and uncorrelation are equivalent. However, \( \hat{N}_1 \) and \( \hat{N}_2 \) usually do not follow a simple distribution if there could be restoration errors. Combining several higher order moments can better evaluate mutual independence.

One advantage of this approximation is that it can be computed based on simple summation over a local neighborhood, which greatly reduces the computational cost.

Therefore, we can design a much more efficient energy term for the cross group regularization in the CFBP. The new energy term enforcing the independence between the estimated streak noise \( \{\hat{N}_i = I_i - \hat{I}\} \) and \( \{\hat{N}_j = I_j - \hat{I}\} \) for sub-reconstruction \( i \) and \( j \) can be defined as:

\[
MI_{i,j}(\hat{N}_i, \hat{N}_j) = \sum_k ||E(h_{1,k}(\hat{N}_i)h_{2,k}(\hat{N}_j)) - E(h_{1,k}(\hat{N}_i))E(h_{2,k}(\hat{N}_j))||^2
\]

(14)

To verify if the proposed independence measurement is correct for the streak noise, we divide the FBP into two sub-reconstructions on a phantom image and calculate the streak noise. The \( MI_{i,j} \) defined in equation (14) (without taking the square) is evaluated within a \( 11 \times 11 \) window at different locations. The histogram is plotted in Figure 1 and shows that this independence measurement works as expected (the value is centered around 0 with small variance).

This regularization term is also extensible based on the available computing power and the expected accuracy of the independence evaluation. For more accurate evaluation of the independence, we can add several terms based on different choices of \( h_1,k() \) and \( h_2,k() \).

If we divide the projections into more than two subgroups, we can use the sum of the pairwise independence to approximate the joint independence across the subgroups. With the new cross group regularization term, the previous MAP framework for the reconstruction can be adapted into the following new form:

\[
\hat{I} = \arg \min_{\tilde{I}} C(\tilde{I}) = \arg \min_{\tilde{I}} (\lambda_0(\hat{I} - \tilde{I})^2 + \lambda \sum_{j=1}^{M-1} \sum_{k=j+1}^{M} MI_{j,k})
\]

(15)

The optimal solution for this new objective function can be found by setting the derivative of the cost function to zero. Since the mutual independence term \( MI_{i,j} \) is dependent on the final reconstruction result \( \hat{I} \), we need to find the optimal solution iteratively. Due to the space limit, we do not include the detailed derivation here. But it is similar to the derivation in subsection 2.3.

We initialize the optimization with a low-pass filtered FBP result (i.e., \( \hat{I}^{(0)} \) = the average of all sub-reconstructions followed by a low-pass filtering). To understand how our new reconstruction method refines this initial guess, we inspect two types of regions. First, for the smooth region where there are no sharp structures, the low-pass filtered FBP result provides noise-free reconstruction. The cross group regularization term is at its minimum (i.e., estimated streak noise is independent) and there is no need to refine these regions. Second, for regions that have sharp structures and streak noise, low-pass filtering not only removes the streak noise, but also blurs the sharp structures. The blurred structures are restoration errors and will increase the cross group mutual dependence. To reduce the dependence, we need to update the estimated reconstruction to include the common signal in all sub-reconstructions while suppressing the uncorrelated components. This pro-
cess is best illustrated in Figure 2(e).

The proposed approximated independence measurement in equation (14) significantly reduces the computational cost of the new energy term. It is worth noting that the computational cost of building all sub-reconstructions is exactly the same as the traditional FBP, which is \(O(N^3)\) if \(N\) is the number of pixels in one projection. With the new independence measurement, the optimization procedure has computational cost of \(O(N^2)\). Hence the CFBP does not cost much more computation than the traditional FBP.

3. Experiments

To validate the new CFBP method, we apply it on a phantom data for quantitative comparisons as well as two real clinical low-dose CT scans under different imaging conditions. The CFBP uses the cost function defined in equation (15) in all experiments. Please note that equation (15) only uses simple noise models and smooth signal constraint with no modeling of edges or corners. When combined with the new cross group independence constraint, the CFBP provides surprisingly good streak noise reduction while preserving the sharp edges and corners.

We compare our CFBP with the traditional MAP method. Since MAP estimation only relies on the generative models. Without explicit edge modeling, MAP estimation significantly blurs the sharp boundaries. For reasonable results, we have to add edge modeling as in [11, 12]:

\[
\hat{I} = \arg \min_I \sum_p (\lambda_1 (I_{1,p} - \hat{I}_p)^2 + \lambda_2 (I_{2,p} - \hat{I}_p)^2
+ \lambda_3 (1 - e_p)(\hat{I}_p - \bar{I}_p)^2 + \lambda_4 e_p)
\]

where \(p\) indicates the pixel’s coordinate. The cost function is a sum over all pixels. \(e_p\) is the edge process which is 0 or 1. When \(e_p = 1\), the smoothness constraint is disabled and hence preserves the sharp edges. To prevent all the pixels from being treated as edges, there is a penalty \(\lambda_4\) for \(e_p = 1\). The MAP restoration quality depends heavily on the accuracy of the edge estimation.

3.1. Quantitative comparisons

To have a quantitative evaluation of our reconstruction algorithm, we conduct a comparison on a 258 \(\times\) 258 phantom image with 120 projections. The phantom image is shown in Figure 2(a). The projection data were corrupted by additive white Gaussian noise of variance 1. Due to the strong contrast edges in the phantom, traditional FBP suffers significant streak noise (Figure 2(b)). With some careful tuning of the parameters, the MAP method can significantly reduce the streak noise for large parts of the image. However, since the streak noise is non-stationary, we have to tune carefully for each image to achieve a good balance between good detection of the true edges and less false detection caused by streak noise. The tuned edge detection result is shown in Figure 2(c). We can see that some strong streak noise is still detected as true signal and hence not reduced while some weak structures begin to get mis-detection and hence blurred. The final MAP reconstruction result is shown in Figure 2(d).

The CFBP requires not much tuning. We divide the projections into three groups and use the low-pass filtered FBP result as the initial guess to start the optimization. For the smooth signal part, the initial guess is quite good because the streak noise is significantly reduced by the low-pass filter. However, the low-pass filtering dramatically blurs the sharp signal and cause restoration error around the sharp edges and corners. The MI term successfully detects the restoration errors (i.e. the blurred structures) with no false detection as shown in Figure 2(e) (white means high cross group dependence hence indicating blurred structures in the initial guess). It is worth noting that the MI term is estimated within a neighborhood (11 \(\times\) 11 in the experiment) and hence cover a bit larger region than the true boundaries. But it does not affect the sharpness of the restored structures because the MI term will be reduced only when the restoration error within the neighborhood is minimized. In the final result shown in Figure 2(f), the streak noise is significantly removed while the sharp weak structures are preserved.

For a quantitative analysis, the results from different methods are compared with the original phantom. To focus on the weak signal in the middle gray levels, we exclude the dark outer parts and the bright white regions when calculating the mean square error and the signal to noise ratio (SNR = 10 log(signal power / noise variance)). The values are shown in Table 1. The new algorithm improves the SNR by 2dB more even compared with the fine-tuned MAP estimation because of the non-stationarity of the streak noise.

<table>
<thead>
<tr>
<th>Table 1. Quantitative comparisons:</th>
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<tr>
<td>Method</td>
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<tr>
<td>Mean Square Error</td>
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<td>SNR</td>
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3.2. Clinical low-dose CT scans

We also test on clinical low-dose CT scans obtained under different imaging settings. The first dataset is acquired with higher X-ray exposure (160mAs) with fewer projections (100 projections with 367 detectors on each projection). The photon noise in the sinogram is not significant. However, due to the small number of projections, the streaks in the reconstructed images are much wider and have more low-frequency components. The second dataset is acquired with lower X-ray dosage (55mAs) but much larger number of projections (580 projections with 1344 detectors on each projection). The photon noise becomes more significant and is amplified and back projected, gener-
Figure 2. Quantitative comparisons on phantom image: (a) Original image; (b) FBP reconstruction from all projections (SNR = -12.7dB); (c) Edge map in the MAP reconstruction; (d) MAP Reconstruction (SNR = 3.55dB); (e) Mutual independence term in the 1st iteration; (f) CFBP result (SNR = 5.46dB).

ating strong high frequency streaks. The proposed CFBP algorithm generalizes well to very different imaging settings.

In the first data set, FBP yields a reconstruction with apparent streak noise. The wider streaks can be clearly seen, especially in the lower right part of the image as shown in Figure 3(a). In the proposed CFBP, the mutual independence analysis successfully captures the reconstruction error around the sharp boundaries caused by the low-pass filtering in the initial guess as shown in Figure 3(b). During the minimization of the mutual information term, the final reconstruction result in (Figure 3(c)) shows significant improvement, where the streak noise is significantly removed while sharp features are well preserved.

The second dataset is shown in Figure 4, which suffers strong high frequency streak noise due to the low dosage of X-ray. Figure 4(a,b) illustrates the whole reconstructed image as well as an enlarged part of the FBP result. The high frequency streak noise can be better seen in the zoomed-in view. The CFBP handles it very well using the same parameters (e.g., the neighborhood size for calculating the mutual information) as the first dataset. The results are shown in Figure 4(c,d). From the enlarged image, we can see the strong reduction of the streak noise and good preservation of the weak structures inside the soft-tissue regions.

4. Conclusion

In this paper, we propose a novel multi-image based restoration method for image reconstruction from projections. A detailed inspection on the generalization issues of traditional Bayesian frameworks is given, focusing on the challenging scenarios where accurate models are not readily available. We demonstrate that carefully designed mutual independence analysis on multiple images can effectively improve the Bayesian frameworks when dealing with the challenging streak noise. Efficient approximation of the mutual independence is also proposed for fast energy optimization. Quantitative comparisons on phantom data and evaluation on clinical low-dose CT scans demonstrate the advantages and effectiveness of our new approach.

References


Figure 3. Clinical low-dose CT scan 1: high X-ray dosage with restricted number of projections (100 projections) causes wide streaks in the reconstruction: (a) FBP reconstruction; (b) Mutual independence term in 1st iteration; (c) CFBP reconstruction result.

Figure 4. Clinical low-dose CT scan 2: low dose X-ray with more projections (580 projections) causes high frequency streaks which can be better seen in the zoomed-in view. (a) FBP reconstruction; (b) Zoom-in of (a); (c) CFBP reconstruction result; (d) Zoom-in of (c).


