Linear Turbo Equalization for Parallel ISI Channels

Jill Nelson, *Student Member, IEEE*, Andrew Singer, *Member, IEEE*, and Ralf Koetter, *Member, IEEE*

***Abstract*—We propose a method for exploiting transmit diversity using parallel independent intersymbol interference channels together with an iterative equalizing receiver. Linear iterative turbo equalization (LITE) employs an interleaver in the transmitter and passes *a priori* information on the transmitted symbols between multiple soft-input/soft-output minimum mean-square error linear equalizers in the receiver. We describe the LITE algorithm, present simulations for both stationary and fading channels, and develop a framework for analyzing the evolution of the *a priori* information as the algorithm iterates.**

***Index Terms*—Belief propagation, decoding, equalizers, intersymbol interference (ISI), iterative methods.**

I. Introduction and Previous Work

Due to the prevalence of intersymbol interference (ISI) channels in a variety of communications systems, much effort has been devoted to developing effective and computationally inexpensive ways to equalize them. The performance of suboptimum equalization schemes can be improved by incorporating soft-input/soft-output (SISO) devices. We propose linear iterative turbo equalization (LITE) [1], an iterative algorithm for combating ISI on multiple independent channels. Data is randomly interleaved prior to transmission over each channel; the receiver employs a SISO minimum mean-square error (MMSE) equalizer for each channel and passes soft information among the equalizers until convergence. Such a scheme could be easily incorporated in an automatic repeat request (ARQ) communications protocol, where packets received with poor signal-to-noise ratio (SNR) are requested for retransmission, by simply permuting the original bit sequence before retransmission.

The LITE algorithm is inspired by the structure of turbo codes [2] and turbo equalization [3]. Since the introduction of these schemes, many iterative algorithms for equalization alone have been proposed [4]–[9]. The LITE algorithm differs from turbo equalization in its use of MMSE rather than maximum *a posteriori* (MAP) equalizers and its methods for using soft information to determine the equalizer coefficients. Though much work exists in the area of multiple-input/multiple-output (MIMO) equalization, it generally focuses on scenarios in which the multiple channels are correlated [10]–[12], while the LITE algorithm is designed to exploit the case in which the channels are independent. This transmission strategy has also been considered in [7], [9], and [13], but with more complex MAP decoding methods.

In [14], density evolution has been used to analyze the performance of belief-propagation decoders for low-density parity-check (LDPC) codes. We use a similar method to study the performance and convergence behavior of the LITE algorithm as a function of the soft information exchanged among equalizers. Its simple structure makes the LITE algorithm an ideal model for examining the role of soft information in a variety of schemes employing SISO MMSE equalizers.

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The authors are with the Coordinated Science Laboratory and the Department of Electrical and Computer Engineering, University of Illinois at Urbana-Champaign, Urbana, IL 61801 USA (e-mail: jknelso1@uiuc.edu; acsinger@uiuc.edu; koetter@uiuc.edu).

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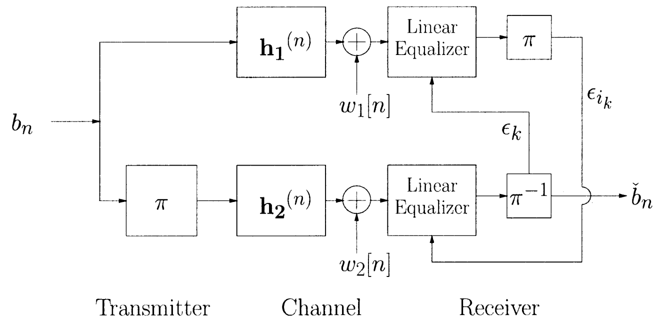

Fig. 1. Equivalent baseband discrete-time channel model and LITE block diagram. Interleaving is represented by π.

II. Channel Model

We consider the channel model shown in Fig. 1. The soft information for bit b_k is denoted by ϵ_k and referred to as the prior on b_k. The data is segmented into blocks, $\{b_n\}_{n=1}^N$, and randomly interleaved prior to transmission over each channel. As a result of this interleaving, the priors generated by one equalizer can be assumed to be independent of those generated by the other equalizer for a certain number of iterations. Hence, priors can be exchanged between equalizers without the creation of short feedback cycles. We assume that the data is binary phase-shift keying (BPSK)-encoded, $\mathbf{h}_1$ and $\mathbf{h}_2$ are discrete-time finite impulse response (FIR) ISI channels, and uncorrelated additive white Gaussian noise (AWGN) $\{w_i[n]\}_{n=1}^N$ is present in each channel. The two channel outputs can be written as

$$r_1[n] = \sum_{k=-L_1}^{L_2} h_1[k] b_{n-k} + w_1[n] \tag{1}$$

$$r_2[n] = \sum_{k=-E_1}^{E_2} h_2[k] \tilde{b}_{n-k} + w_2[n] \tag{2}$$

where b_n is the input symbol stream, $\tilde{b}_n$ is the interleaved symbol stream, and $L_1 + L_2 + 1$ and $E_1 + E_2 + 1$ are the lengths of the channels.

III. The LITE Algorithm

The SISO equalizers in the LITE algorithm take as input both a received sequence $\{r_n\}_{n=1}^{N}$ and prior probability distributions over each of the transmitted bits. Each equalizer then produces new priors on the symbols. Such priors are passed between the linear equalizers until a convergence criterion is reached. The received vector can be written in matrix form as

$$\mathbf{r}_n = \mathbf{H}\mathbf{b}_n + \mathbf{w}_n \tag{3}$$

where $\mathbf{H}$ is the convolution matrix whose rows are shifted versions of $\mathbf{h}$, and $\mathbf{b}_n$ and $\mathbf{w}_n$ are vectors of transmitted bits and noise samples, respectively. Given the received vector $\mathbf{r}_n$, the output of the equalizer can be expressed as

$$\check{b}_n = \mathbf{c}^{(n)T}\mathbf{r}_n \tag{4}$$

where $\mathbf{c}^{(n)}$ denotes the equalizer coefficients at time n. With this model, the mean-squared error (MSE) of a symbol estimate $\check{b}_n$ is given by $\mathcal{E}(\check{b}_n) = E_{b,w}\{|b_n - \check{b}_n|^2\}$, where the notation $E_{b,w}$ indicates an expectation over the distribution of the symbols $\{b_n\}_{n=1}^{N}$ and the noise $\{w_n\}_{n=1}^{N}$.

Traditional MMSE linear equalization assumes that the transmitted symbols are independent and identically distributed (i.i.d.) with all values equally likely, thus producing time-invariant equalizer coefficients. In the LITE algorithm, however, priors are available from each equalizer and are included in the error minimization, yielding time-varying equalizer coefficients $\mathbf{c}^{(n)}$. For BPSK, the prior information can be represented with a scalar as $\epsilon_n = \Pr\{b_n = 1\}$. These priors are incorporated into the design of the linear MMSE equalizer by minimizing $\mathcal{E}(\check{b}_n)$ noting that $\{b_n\}_{n=1}^{N}$ are no longer i.i.d. Similar equalizer designs and low-complexity approximations to these designs are developed in [4], [5], [15], and [16].

For the first pass through the equalizer for channel 1, we set $\epsilon_n = 1/2$; for all subsequent iterations, the MMSE linear estimates of b_n produced by one equalizer are mapped into priors and passed to the other equalizer. Assuming the distribution of the symbol estimates is conditionally Gaussian, we write

$$p(\check{b}|b=1) = \frac{1}{\sqrt{2\pi\sigma_b^2}}\, e^{(-(\check{b}-\mu_b)^2)/2\sigma_b^2} \tag{5}$$

and similarly for $p(\check{b}|b=-1)$, where μ_b and σ_b^2 are the mean and variance of the estimate $\check{b}$ conditioned on $b = 1$. The prior ϵ can then be calculated as follows:

$$\begin{aligned}\epsilon = \Pr(b=1|\check{b}) &= \frac{\Pr(\check{b}|b=1)\Pr(b=1)}{\Pr(\check{b})}\\ &= \frac{1}{2}\left(1 + \tanh\left(\frac{\mu_b \check{b}}{\sigma_b^2}\right)\right).\end{aligned} \tag{6}$$

In our analysis of the LITE algorithm, we consider the distribution over the ensemble of the priors, and hence, require a time-invariant approximation to the time-varying quantities μ_b and σ_b^2 [16]. We use

$$\mu_b \approx \frac{1}{2}\left(\frac{1}{N_+}\sum_{n:\,\check{b}_n>0}\check{b}_n - \frac{1}{N-N_+}\sum_{n:\,\check{b}_n<0}\check{b}_n\right) \tag{7}$$

$$\sigma_b^2 \approx \frac{1}{2}\left(\frac{1}{N_+}\sum_{n:\,\check{b}_n>0}\left(\check{b}_n - \mu_b\right)^2 + \frac{1}{N-N_+}\sum_{n:\,\check{b}_n<0}\left(\check{b}_n + \mu_b\right)^2\right) \tag{8}$$

where N_+ denotes the number of positive elements in the block of estimates $\{\check{b}_n\}_{n=1}^{N}$.

Standard assumptions about the independence of the priors as a result of interleaving are applied. For the LITE algorithm to be well behaved [15], we also require that the estimate of a bit b_k not be a function of the prior ϵ_k. Thus, we set ϵ_k equal to $1/2$ when b_k is estimated, i.e., we pass only extrinsic information between the SISO equalizers.

Using recursive updates to compute the equalizer coefficients, the complexity of the LITE algorithm can be reduced to, at most, $O(L^2)$ [4], [15], where L is the length of the channel; in contrast, MAP and maximum-likelihood sequence (MLS) detection are exponential in the channel length. In addition, the complexity of the MMSE equalizers employed in the LITE algorithm does not increase with higher order symbol constellations [15], while the complexity of MAP detection grows as S^L, where S is the size of the symbol constellation.

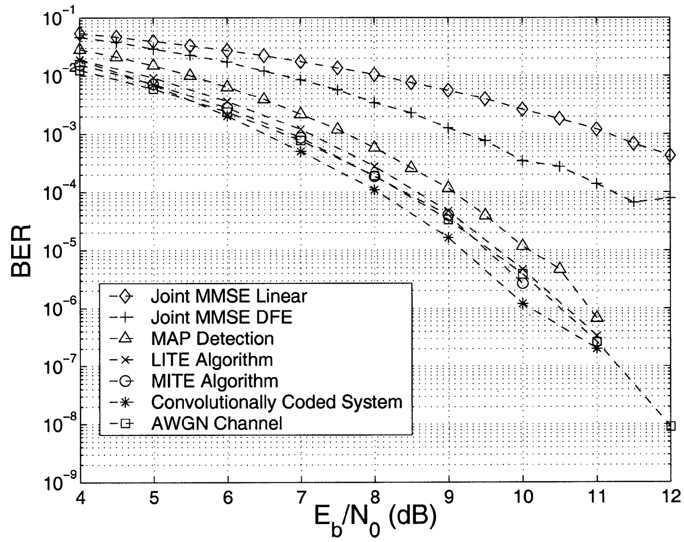

Fig. 2. Performance of a range of equalization schemes on stationary channels.

IV. Performance of the LITE Algorithm

The LITE algorithm has been simulated for $\mathbf{h_1} = \mathbf{h_2} = [0.1474\ \ 0.8847\ \ 0.4423]^T$. One thousand blocks of length of $N = 1000$ symbols have been processed with a five-tap equalizer. Fig. 2 shows the performance results for the LITE algorithm as well as for a variety of other equalization schemes. The plot reveals that the LITE algorithm attains performance gains of at least 3 dB over both joint MMSE and decision-feedback

TABLE I
CHANNEL PARAMETERS AND POWER DELAY SPECTRUM FOR HILLY TERRAIN FADING CHANNEL MODEL WITH 12 INDEPENDENTLY RAYLEIGH FADING TAPS

Mobile speed	100 km/hr
Transmission frequency	900 MHz
Symbol period	3.692 μs
Channel length	7 taps
Channel sampling period	4.615 ms

Path	1	2	3	4	5	6	7	8	9	10	11	12
Relative Time (μs)	0.0	0.1	0.3	0.5	0.7	1.0	1.3	15.0	15.2	15.7	17.2	20.0
Relative Power (dB)	-10	-8	-6	-4	0	0	-4	-8	-9	-10	-12	-14

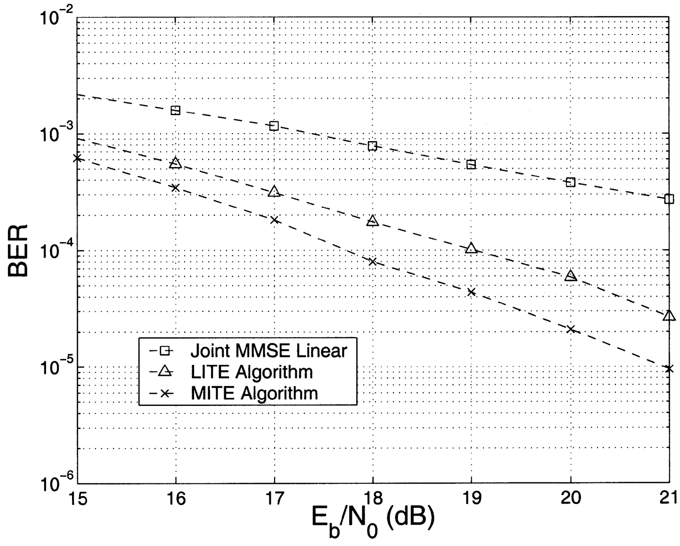

Fig. 3. Performance of joint MMSE linear, LITE, and MITE equalizers on independently Rayleigh fading channels.

equalization (DFE) and slight gains over MAP detection without interleaving. In addition, the LITE algorithm performs up to 4 dB better than a repetition code and within 0.5 dB of a convolutionally coded system with generator sequences $\mathbf{g_1} = [101]$ and $\mathbf{g_2} = [111]$. Finally, the LITE algorithm nearly achieves the bit-error rate (BER) of the AWGN performance curve and is within 0.25 dB of the performance of MITE, the turbo-equalization structure using MAP detectors instead of MMSE equalizers, for $E_b/N_0 > 5$ dB.

To explore wireless applications, we simulate the LITE algorithm for $\mathbf{h}_1$ and $\mathbf{h}_2$ independently Rayleigh fading multipath channels. To generate the channels, a global system for mobile communications (GSM) hilly terrain model with 12 multipath delays [17] was used; the parameters of the model are given in Table I. The resulting channels were baud sampled yielding equivalent discrete-time baseband responses of roughly seven nonzero taps. We assume a block fading scenario and consider 300 blocks of 500 symbols each. A nine-tap equalizer is used for both joint MMSE equalization and the LITE algorithm. Fig. 3 shows that the LITE algorithm performs at least 2 dB better than joint MMSE equalization for BER $< 10^{-3}$ and at least 4 dB better than joint MMSE equalization for BER $< 10^{-4}$. In addition, the LITE algorithm performs within 1.5 dB of the MITE algorithm.

V. EVOLUTION OF PRIORS IN THE LITE ALGORITHM

In this section, we develop a framework for analyzing how the priors change as the LITE algorithm iterates. Since the priors are functions of the received data, they are random variables and can be viewed as originating from a single marginal distribution. For simplicity, we assume that this distribution is discrete. Given a set of priors chosen according to an initial probability mass function (PMF) as input to the LITE algorithm, we examine the evolution of this PMF as the algorithm iterates.

We refer to the vector of input bits $\mathbf{b}_n$ (the bits that affect the estimate $\check{b}_n$ of the bit b_n) as a bitword. For a given bitword $\mathbf{b}_n$ and an associated vector of priors $\boldsymbol{\epsilon}_n$ as inputs, the estimate $\check{b}_n$ given by one iteration of the LITE algorithm is Gaussian distributed due to AWGN present in the channel. The mean and variance of this distribution can be written as

$$\mu(\check{b}_n|\mathbf{b}_n, \boldsymbol{\epsilon}_n) = \mathbf{c}^{(n)T}\mathbf{H}\left(\mathbf{b}_n - E[\mathbf{b}_n]\right) \quad (9)$$

$$\sigma^2(\check{b}_n|\mathbf{b}_n, \boldsymbol{\epsilon}_n) = \sigma_w^2\mathbf{c}^{(n)T}\mathbf{c}^{(n)} \quad (10)$$

where $\sigma_w^2 = E\{|w_n|^2\}$. We consider a five-point PMF with values equally spaced between 0 and 1. Since the cases of $b_n = 1$ and $b_n = -1$ (where b_n is the symbol to be estimated) are symmetric, we study only the case in which $b_n = 1$ and the other elements of the bitword $\mathbf{b}_n$ take all possible combinations of $+1$ and -1. We calculate the mean and variance of the Gaussian distribution of $\check{b}$ that results for each possible combination of bitword and input priors, $(\mathbf{b}_i, \boldsymbol{\epsilon}_j)$ and map the distributions to PMFs. To calculate the output PMF for a given iteration, each PMF is weighted according to the probability that the $(\mathbf{b}_i, \boldsymbol{\epsilon}_j)$ pair that generated it occurs. For the first iteration, no prior information is available, i.e., $\Pr\{\epsilon = 0.5\} = 1$; for all following iterations, we compute $\Pr\{(\mathbf{b}_i, \boldsymbol{\epsilon}_j)\} = \Pr\{\boldsymbol{\epsilon}_j|\mathbf{b}_i\}\Pr\{\mathbf{b}_i\}$ for each $(\mathbf{b}_i, \boldsymbol{\epsilon}_j)$. Since $(b_i[k], \epsilon_j[k])$ are assumed to be independent of all $(b_i[l], \epsilon_j[l])$ for $k \neq l$, we can write

$$\Pr\{\boldsymbol{\epsilon}_j|\mathbf{b}_i\} = \prod_{k=1}^{P}\left(\Pr\{\epsilon_j[k]|b_i[k]\} = \begin{cases}\Pr\{\epsilon_j[k]\} & b_i[k] = 1\\ \Pr\{1-\epsilon_j[k]\} & b_i[k] = -1\end{cases}\right) \quad (11)$$

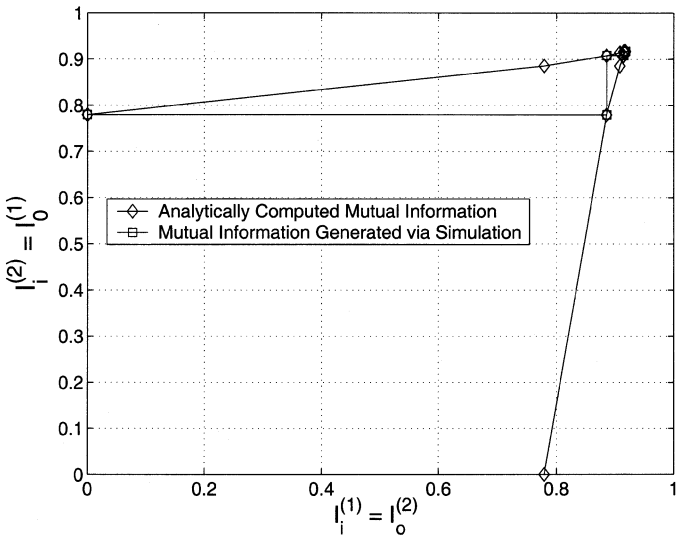

Fig. 4. EXIT chart generated via PMF analysis and mutual information computed from LITE simulation over five iterations. I_i and I_o denote mutual information at the input and output of a device, respectively; the superscript indicates which equalizer is being considered.

where P is the length of $\mathbf{b_i}$. All bitwords are assumed to be equally likely, so $\Pr\{\mathbf{b}_i\} = 2^{-P+1}$ for any $\mathbf{b}_i$. By total probability, the output PMF is a weighted sum of the individual PMFs

$$\Pr\{\epsilon = a\} = \frac{1}{2^{P-1}} \sum_{i=1}^{2^{P-1}} \sum_{j=1}^{5^{P-1}} \Pr\{\epsilon = a|\mathbf{b}_i, \boldsymbol{\epsilon}_j\} \Pr\{\mathbf{b}_i, \boldsymbol{\epsilon}_j\}, \quad a \in \{0, 0.25, 0.5, 0.75, 1.0\}. \tag{12}$$

We have analyzed the evolution of the PMF of the priors for $\mathbf{h_1} = \mathbf{h_2} = [0.1474 \ 0.8847 \ 0.4423]^T$ and an equalizer of length five. For $E_b/N_0 = 6$ dB, the results show that the weight at each point of the PMF is nearly constant after only two iterations, indicating that the PMF appears to converge.

To explore the prediction of BER from PMFs, we simulate the LITE algorithm for $\mathbf{h_1}$ and $\mathbf{h_2}$ as given above. We quantize the priors to the five-point PMF and compute the conditional mean and variance analytically, i.e.,

$$E[\check{b}_n|b_n = 1] = \sum_{i,j} \Pr\{(\mathbf{b}_i, \boldsymbol{\epsilon}_j)\} E[\check{b}_n|(\mathbf{b}_i, \boldsymbol{\epsilon}_j)] \tag{13}$$

$$\mathrm{var}(\check{b}_n|b_n = 1) = \sum_{i,j} \Pr\{(\mathbf{b}_i, \boldsymbol{\epsilon}_j)\} E[\check{b}_n^2|(\mathbf{b}_i, \boldsymbol{\epsilon}_j)] - \left(E[\check{b}_n|b_n = 1]\right)^2 \tag{14}$$

to more closely match the structure of the analysis. Approximating BER from the PMF analysis as $\mathrm{BER} \approx \Pr\{\epsilon = 0\} + \Pr\{\epsilon = 0.25\} + 0.5\Pr\{\epsilon = 0.5\}$, the results show a clearly linear relationship between the approximated and simulated BER. For $E_b/N_0 = 7$ dB, the estimated BER is within 5% of the BER in practice.

Extrinsic information transfer (EXIT) charts, introduced by ten Brink in [18], track the evolution of mutual information between the transmitted data and the priors as an algorithm iterates. While EXIT charts are typically constructed empirically, the PMF analysis described above allows us to compute EXIT charts analytically. For each iteration, we compute the mutual information between the transmitted bits and the output PMF generated in that iteration. We plot in Fig. 4 the mutual information at each iteration of the simulation along with the analytically generated EXIT chart, both for $E_b/N_0 = 7$ dB. The mutual information computed via simulation nearly exactly follows the evolution predicted by the EXIT charts generated from the PMF analysis, indicating that the analytically computed EXIT chart can be used to predict the maximum mutual information that can be achieved and the number of iterations required to reach that value.

VI. Conclusion

We have presented an iterative equalization algorithm for multiple ISI channels which passes soft information between multiple MMSE equalizers. Simulation results show that the LITE algorithm performs significantly better than conventional low-complexity methods and nearly as well as MAP estimation. We have also developed a framework for analyzing the evolution of the priors in the LITE algorithm and discussed its application to BER estimation and EXIT chart generation.

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