A DFE Coefficient Placement Algorithm for Sparse Reverberant Channels

Michael J. Lopez and Andrew C. Singer

***Abstract*—We develop an automated algorithm for determining the number and sparsely supported locations of coefficients in a multi-element array decision-feedback equalizer based on an estimated channel response. We aim for robustness to a wide variety of possible channel conditions, especially through taking advantage of the interplay between the MMSE-optimal feedforward and feedback filters.**

***Index Terms*—Adaptive equalizers, decision feedback equalizers, reduced order systems, time-varying channels, underwater acoustic communication.**

I. Introduction

THE adaptive decision-feedback equalizer (DFE) has become a common and useful tool for high-speed digital communications over time-varying frequency-selective channels. However, channels with significant reverberation of, say, greater than one hundred symbol periods present considerable practical challenges, as they may require filters with several hundred coefficients to fully compensate for the channel. Adaptation of such long filters tends to be computationally prohibitive, unresponsive to channel fluctuations, and may require excessive training data to initially learn the equalizer coefficients. Therefore, in this paper, we present a robust, automated tap placement routine which attempts to place filter taps only where necessary, overcoming some of the disadvantages of previous approaches to this difficult problem.

We concentrate on the underwater acoustic channel as an example of an adverse environment for digital communication; however, similar issues arise in other environments with large delay spreads and rapid fading, such as HF over-the-horizon radio communication [1], [2]. Horizontal underwater acoustic channels may exhibit a delay spread of 80 ms or longer, may or may not contain resolvable multipath arrivals, and tend to vary considerably over the several seconds of a typical data packet (see Fig. 1). Spatial diversity combining is often necessary to combat high noise levels. Despite these difficulties, recent advances in underwater acoustic communication using adaptive equalization and phase compensation [3] have shown the viability of high data rate, coherent digital communication techniques over long ranges in these adverse conditions.

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II. Sparse DFE

A DFE can be thought of as equalizing a channel in two steps: first, a feedforward section (linear filter) shapes the overall response appropriately and attempts to make the intersymbol interference (ISI) causal, and then the feedback of sliced (quantized) outputs cancels postcursor ISI. We can express the output of a (nonsparse) DFE as follows:

$$\hat{d}[n] = \sum_{k=-L_1}^{L_2} a[k]r[n-k] + \sum_{k=1}^{M} b[k]\tilde{d}[n-k], \qquad (1)$$

where

$r[n]$ baseband received data;

$\hat{d}[n]$ pre-sliced decision variables;

$\tilde{d}[n]$ sliced (final) decision variables.

Given a channel impulse response $h[n]$, symbol variance σ_d^2, white additive noise variance N_0, and a desired filter tap support, the minimum mean-square error (MMSE) optimal DFE coefficients can be computed by

$$[a[-L_1], \ldots, a[L_2]]^{\mathrm{T}} = R^{-1}\mathbf{p}$$
$$b[n] = \sum_{m=-\infty}^{\infty} h[m]a[n-m], \quad n = 1, \ldots, M$$

where $\mathbf{p} = \sigma_d^2 [h^*[L_1] \ldots h^*[-L_2]]^{\mathrm{T}}$ and R is the output correlation matrix with the appropriate postcursor intersymbol interference terms removed [4]. The "estimated mean-square error" (EMSE) which would arise from the channel response estimate $h[n]$ is given by,

$$E[(d[n] - \hat{d}[n])^2 | h[n]] = \sigma_d^2 - \mathbf{p}^{\mathrm{H}} R^{-1}\mathbf{p}, \qquad (2)$$

where A^{H} denotes the Hermitian transpose of A. Extensions to fractionally-spaced and multi-element array systems are straightforward; see, for example, [4] for details.

The goal of a sparse equalizer is to set as many of the values of $a[n]$ and $b[n]$ in (1) as possible to zero, while still correctly decoding the data or achieving some prescribed MSE. There is no practical, straightforward method known to determine this filter support. Previous methods for selecting feedforward taps include a suboptimal search involving an approximation of SNR at the slicer input [5], [6], selecting significant arrivals by thresholding the channel impulse response estimate directly [7], or by an *ad hoc* choice of contiguous taps around the central arrival [8], [9]. The papers in [10] and [11] also chose contiguous support, with a constant number of taps selected so that typical channels can be equalized. Feedback support for these algorithms is either unspecified, optimally long given the number of taps in the feedforward support and channel impulse response

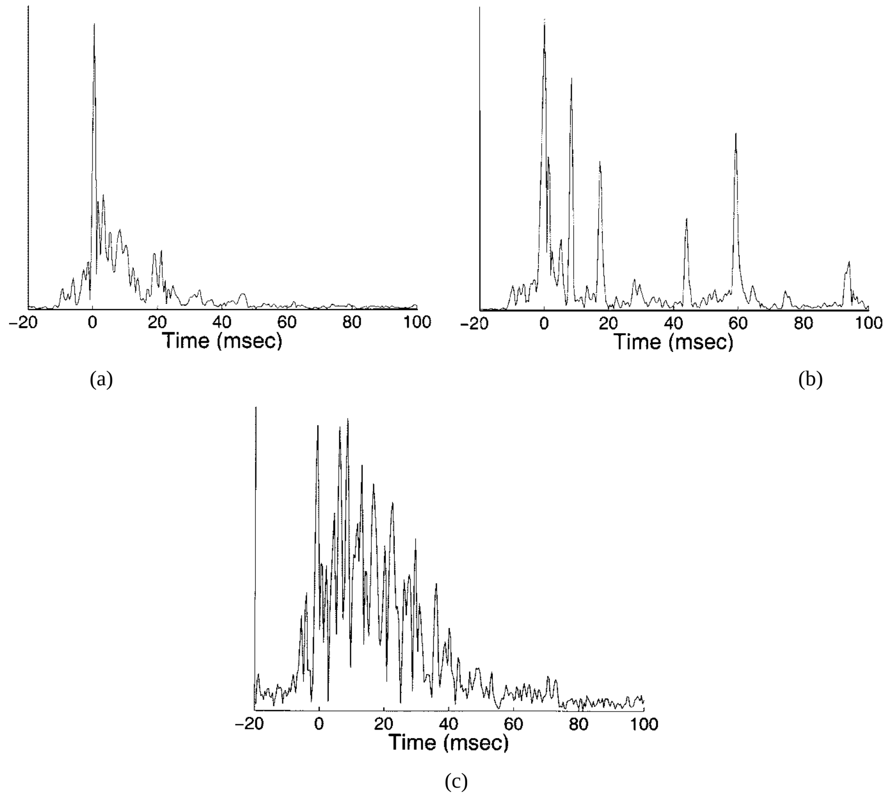

Fig. 1. Typical measured equivalent baseband channel responses for the three data sets. (a) Short response, (b) Long, sparse response, (c) Long, nonsparse response.

estimate, or determined by thresholding either these optimal feedback coefficients or the channel impulse response.

In light of these methods, we now describe a number of issues considered in the development our tap placement algorithm. First, the feedforward support has traditionally been determined independently of the feedback selection or by assuming perfect (nonsparse) feedback. However, since their effects on the resulting DFE are highly coupled, the feedforward and feedback tap placements should be jointly determined. For example, if there are a large number of feedforward coefficients, a later multipath arrival may be nulled without the need of additional feedback. On the other hand, placing a small amount of feedback around the position of strong ISI may allow the feedforward filters to concentrate effort elsewhere and thus achieve a lower MSE. To take this into account but still maintain reasonable complexity, we propose an exchange-type algorithm, which alternates between updating feedforward and feedback tap placements. At each stage, simple rules are used to place additional taps until a performance criterion is met.

Furthermore, in these previous methods, the number of sparse taps is either a constant parameter or will be indirectly determined based on threshold levels. However, the selection of a single threshold or number of taps becomes problematic for a robust, automated routine expected to deal with a variety of channels and multi-element array data. More generally, such thresholding methods often select taps based on the energy of either the channel response or of the nonsparse tap amplitudes. While thresholding can be connected with a least-squares approximation to the frequency response of the nonsparse equalizer, via Parseval's Theorem, the underlying goal of an MMSE equalizer should be to decrease the error at the slicer input. Therefore, we instead base our stopping criteria on the EMSE.

Unfortunately, the EMSE from (2) does not correspond to the error in actual operation due to effects such as imperfect channel estimation, unmodeled noise propagation which increases with filter length, and temporal changes in the channel response. Since the filter taps must be estimated from the data as they are applied to the received symbols, the performance of such adaptive equalization algorithms are often sensitive to the selected number of taps used, or the model-order of the equalizer. In fact, as the number of filter coefficients increases, the adaptive estimation problem eventually is overparameterized and the MSE in actual operation will begin to increase as more parameters are added [12]. This estimation performance penalty with increasing model order can be viewed as an expression of Rissanen's minimum description length (MDL) for estimation of Gaussian processes [13]. To take this into account in our automated tap placement routine, we add a model order penalty term to the EMSE that is proportional to Risannen's penalty for Gaussian data, i.e., $p\log(n)/2n$, where $p = KL$ is the number of parameters, K is the number of array elements, L is the number of taps per array element, and n is the data length. This penalty is also equivalent to the expected excess mean-squared estimation error that would be achieved from a recursive-least squares algorithm with Gaussian data [14]. For a particular K and n, we therefore look to minimize an expression of the form,

$$\gamma = \text{EMSE} + \beta L \tag{3}$$

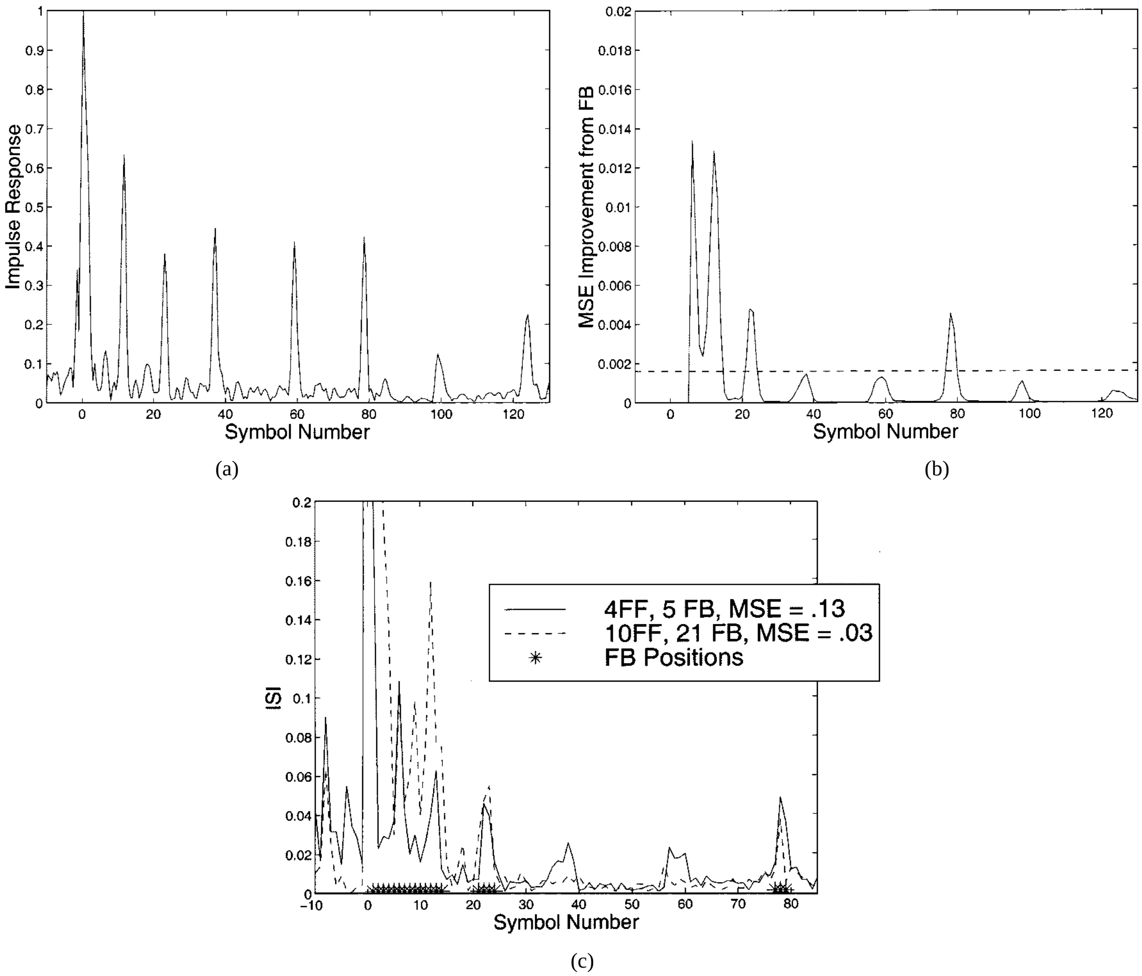

Fig. 2. Illustration of parts of the algorithm for sparse, four element array data. (a) Impulse response est. for array element 1, (b) EMSE improvement by adding feedback, (c) ISI before and after the main routine. For the dotted curve, ISI at the ('$*$') positions will be canceled by feedback.

where β is a constant. Note if only feedforward or feedback coefficients are being added at each stage of the algorithm, (3) can be interpreted as a stopping criterion.

III. Filter Tap Optimization Algorithm

Our tap placement algorithm consists of iterations which alternate between updating feedforward and feedback taps. For the feedforward tap stages, we choose to concentrate the support in a contiguous block centralized around the main arrival (as in [8], [9]), and add taps until the measure in (3) is minimized. Such filters attempt to concentrate energy from the main arrival, while steering ISI toward the positions of the feedback taps where it can be cancelled. The same number of taps is used for each array element. Note that the EMSE values computed here take into account the feedback tap support from the previous iteration.

Different simplifications are made for the feedback stages. Close to 100 sparse feedback taps may be necessary to equalize the data, making it impractical to recompute the EMSE after placing each tap. We instead take the feedforward and feedback support from the previous iteration and compute the potential EMSE improvement of adding one additional feedback tap at any position. Taps are added where this improvement is greater than some threshold δ. In this way, our criterion can again be interpreted as minimizing an objective function similar to (3).

With these simplifications, our tap placement algorithm can be summarized by the following steps:

i) *Ramp up*: Add initial feedforward and feedback taps until some loosely-set noise margin is met. For QPSK data, $\text{EMSE} = 1/\sigma_d^2$ works fine.
ii) *Feedback*: Place additional feedback taps where they will improve EMSE by at least an amount δ.
iii) *Feedforward*: Increase L, the number of feedforward taps per array element, until a minimum is found for γ in (3).
iv) Repeat *Feedback* step.

Including the ramp up of feedforward taps, this essentially consists of two iterations of each (feedforward and feedback) tap placement stage.

Fig. 2 demonstrates the algorithm on four-element array data collected from an environment with a sparse impulse response. The impulse response estimate from the first array element, available from a probe sent prior to the data packet, is shown in Fig. 2(a). Step (i) of the algorithm estimated that 4 feedforward taps per array element and 5 contiguous feedback taps should provide a stable initial configuration for the following stages.

Fig. 2(b) shows a smoothed graph of the EMSE improvement if a single feedback tap were added at any position; taps will be placed where the curve lies above the threshold shown. The potential EMSE improvement of adding feedback taps can be computed efficiently by noting that the R that appears in (2) can rewritten as $C^{\mathrm{H}}C + N_0 I$, where C contains shifted versions of the channel impulse response. Adding a new feedback tap at lag n corresponds to removing a row $\mathbf{c}^{\mathrm{T}}$ of C. Applying the matrix inversion lemma to (2), we arrive at

$$\text{EMSE improvement} = \frac{\sigma_d^2 |b[n]|^2}{1 - \mathbf{c}^{\mathrm{H}} R^{-1} \mathbf{c}}. \tag{4}$$

Note that the EMSE improvement is not just a scaled version of the impulse response in Fig. 2(a), as would be assumed by a method which places taps based solely on thresholding the impulse response estimate.

Taking these new feedback positions into account, step (iii) determines the final number of feedforward coefficients, which came out to 10 per array element for this example. With the feedback positions placed properly, the feedforward coefficients can most efficiently concentrate on lowering the ISI at other locations. Fig. 2(c) shows the ISI from the combined channels and optimal feedforward filters, after steps (i) and (iii). In the latter case (the dashed curve), we see that little attempt is made to reduce ISI at the feedback tap positions (marked '$*$') because it will be later eliminated by feedback. Instead, the precursor ISI, for example, can be decreased dramatically.

The values for the parameters β and δ were determined empirically, but with motivation from the MDL criterion [13] and mean excess estimation error for Gaussian data [14]. If N is the number of data symbols in each packet, then the penalty for the number of parameters under MDL should be $(0.5 \log_2(N))/N = 0.001$ for the data sets described in the following section. The value of δ should be set somewhat less than β, because (4) overestimates the EMSE improvement by not taking into account other feedback taps that have been placed during the same stage. In practice, we found that values of 0.0033 and 0.0016 for β and δ, respectively, worked well for all of our experimental data sets, though the performance was not particularly sensitive to these precise values. Although these parameters were empirically optimized, their dependence on EMSE rather than channel response amplitudes will lead to systems that are more robust, in the number of array elements used and in the types of responses they can equalize, than presently used techniques. If the values were selected using MDL, then for a four-channel data set, we would have selected $\beta = .004$, which is sufficiently close to our empirically set value that performance differences would be negligible.

IV. Experimental Results and Conclusions

The algorithm was tested on QPSK data (with transmitted symbols $\pm 1 \pm j$) sent at 2.5 kb/s in three different experiments conducted in the Narragansett Bay Operating Area (NBOA), on the continental shelf. Typical channel responses for the three sets are shown in Fig. 1. Six-element array data was used to decode the first and third data sets, while the long, sparse responses of the second set were tested with four-element array data. Using the resulting filter support, a 1/2 fractionally spaced, RLS-updated adaptive DFE was run on 6784 symbols, including 512

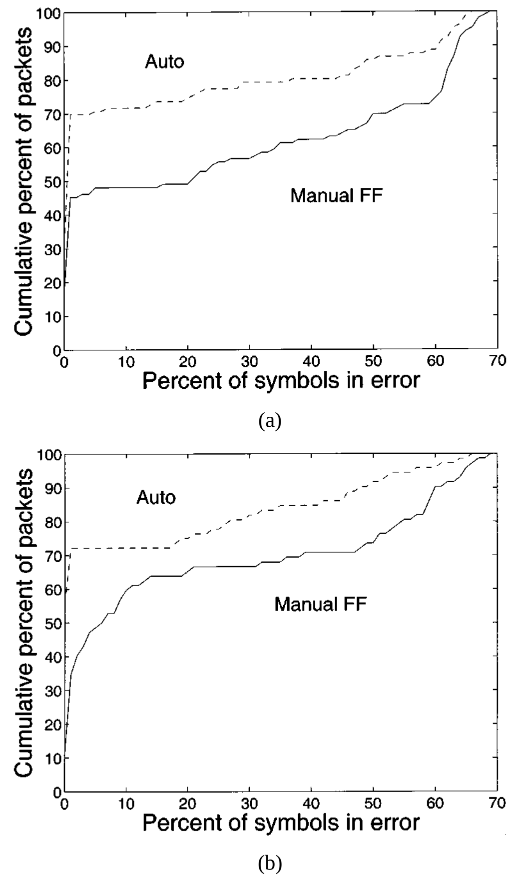

Fig. 3. Cumulative error percentage distributions from running automated and *ad hoc* (manual FF) tap placement routines. (a) Six array elements, data set 3 (106 packets). (b) One array element, data set 2 (72 packets).

training symbols; see [3] for details on the equalizer. For comparison, a tap placement method similar to that used in [9] was also run. Here, feedback taps are selected by channel impulse response thresholding and the number of contiguous feedforward taps per array element is taken as constant for an entire data set, being manually chosen from equalizing a few representative packets.

Both algorithms were successfully able to equalize the first two data sets, using similar numbers of parameters (averages of 10 feedforward taps per array element and 25 feedback taps) and only requiring adaptive filter updates on about 15% of symbols to maintain a minimum output MSE. However, the automated routine was better able to equalize the most difficult data set, consisting of long, nonsparse channel responses requiring 16 or more feedforward taps per array element and around 60 feedback taps. Fig. 3(a) shows the percentage of packets which achieved a given symbol error rate or better. The improvement was largely facilitated by the new routine's ability to efficiently adjust the length of the feedforward filters based on the difficulty of each individual packet. Furthermore, on this data set, by far the most computationally intensive, the overhead of the tap placement routine was generally below three percent of the floating point operations required by the adaptive equalizer itself.

As an additional test, the second data set was also run with only a single array element, with the results shown in Fig. 3(b). Our routine was able to automatically choose larger filters when necessary and equalized more than half the packets, while the more ad-hoc routine's parameters would have needed to be readjusted to fit each new scenario.

These results indicate a potential robustness of the tap placement algorithm to decoding a diverse class of channel responses and different numbers of diversity array elements. This may arise from taking advantage of the interplay between the feedforward and feedback taps, as well as incorporating multi-element array data through the use of the estimated MSE calculations.

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