TURBO EQUALIZATION WITH AN UNKNOWN CHANNEL

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ABSTRACT

We consider the problem of joint equalization and decoding, using the method of turbo equalization originally developed by Douillard, et al. [3]. In its original form, turbo-equalization requires accurate knowledge of the channel at the receiver. We propose a receiver structure, based on a soft-input Kalman channel estimator, that can operate effectively without accurate channel knowledge and without training data. The resulting joint channel and data estimator is shown to outperform standard turbo equalization based on moderate-length traning data.

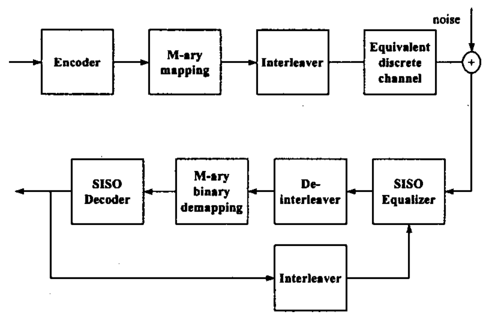

Fig. 1. Turbo equalizer block diagram

1. INTRODUCTION

Since the introduction of turbo equalization in [3], a number of researchers have considered practical implementations of this scheme, e.g. [4, 5], as well as incorporating soft information from decoding into a host of additional communications and signal processing tasks, e.g. [6]. The essential idea behind each of these strategies is to exploit the additional structure imposed on the transmitted data sequence, originally for the purpose of error-control coding, to enhance the performance of other tasks, such as equalization, detection, and signal separation. While a true jointly optimal solution to these various tasks could be undertaken, the computational complexity rapidly becomes insurpassable, when, for example, convolutional decoding is combined with channel equalization or multi-user detection. However, in the same spirit of the original turbo decoding [1] method, by judiciously interleaving the data sequence between operations, and passing soft information between tasks in the form of priors over the data symbols, relatively low complexity approaches can provide significant synergy gains achieving nearly all of the synergy gains that could be obtained from a true joint processing.

In this paper, we approach the problem of joint equalization and decoding of data protected by an error-control code when there is significant uncertainty in knowledge of the channel at the receiver. While the original turbo equalization approach performs extremely well when the channel is known at the receiver, a number of difficulties arise when there is little or no channel information known. One approach to this problem is to consider the channel estimation problem as simply one additional signal processing task to be performed jointly with the data equalization and decoding tasks. For example, this approach is considered in [2], however the resulting complexity of the algorithm rapidly becomes intractable due to the interconnection of the equalization and channel estimation tasks. In this paper, we restructure the channel estimation problem as one of Kalman state estimation, by incorporating the soft information from the decoding process into the statistical description of the channel. One significant benefit of this approach is that it dramatically reduces, if not completely eliminates the need for pilot symbols for channel acquisition. The net result is an algorithm that is robust to a wide variety of uncertanties without substantial training overhead.

The paper is organized as follows. In Section II, we describe the turbo equalization structure for joint equalization and decoding. The proposed receiver structure is then presented in Section III. In Section IV, an algorithm is presented for finding the best linear unbiased channel estimate based on the received data and the output of the decoding algorithm. In Section V, a performance comparison is provided for this overall system versus standard turbo equalization, as well as a more standard channel estimator, based on hard decisions from the decoding process.

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2. SYSTEM OVERVIEW

Consider the base band transmission system depicted in Fig. 1, where the source bit stream $\{s(n)\}$ is encoded with a rate R_c convoulutional code of memory L_c bits. The encoded symbols are then interleaved to randomize the time-axes as seen by the decoder, and therfore reduce the influence of error bursts from the equalizer to the input of the channel decoder. The interleaved code bits $\{a(n)\}$ are M-ary mapped and transmitted over a frequency selective channel. Usually, the overall modulator-channel-demodulator is modeled with a time varying finite impulse response filter, $h_{n,k}$ of length L, and then the baud-sampled received signal $r(n)$ is represented as

$$r(n) = \sum_{k=0}^{L-1} h_{n,k} b(n) + w_0(n), \quad n \geq 0 \tag{1}$$

where $b(n)$ denotes the transmitted symbols which are encoded and M-ary mapped, and w_0 is additive white Gaussian noise with zero mean and variance of $\frac{N_0}{2}$. A typical model used in wireless communications is that each channel tap, corresponding to a different multi-path arrival, is an uncorrelated wide sense stationary random process, yielding the standard uncorrelated scattering (WSSUS) model. We will exploit this model later for channel estimation.

The turbo equalization receiver contains an equalizer and decoder, each of which can make optimal use of additional soft information, in the form of a prior over the observed data symbols. On the first pass, the equalization algorithm is a standard MAP decoder, or BCJR algorithm for the channel. The resulting outputs can be viewed as probabilities that each data symbol took on each possible value, i.e. a probability mass function (pmf) for each data bit $b(n)$. After de-interleaving, these prior probabilities are then passed to the channel decoder, which makes optimal use of them when decoding received data stream - the decoding is optimal in an MAP sense based upon the received prior probabilities alone. This is approximate, in that the observed data, and any potential correlation between samples has been assumed removed from the interleaving process. The decoder then produces a similar set of soft-information, which, when re-interleaved, are passed back to the MAP equalizer. On this and subsequent iterations, the MAP equalizer behaves as a soft-input/soft-output (SISO) device, again making optimal use of this new soft information (ignoring possible temporal dependence) to produce a new soft output. These SISO devices jointly exchange the soft information until convergence.

3. THE PROPOSED RECEIVER

In order to exploit the potential performance of turbo equalization, a receiver requires a high quality channel estimate.

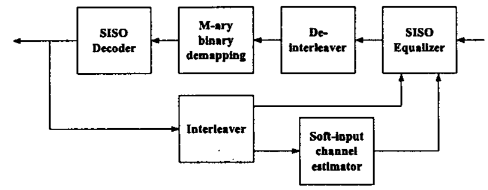

Fig. 2. Proposed receiver structure

Thus, this eventually necessitates the use of a considerable portion of the data frame as a training sequence. Meanwhile, this redundancy in the data frame ultimately limits system throughput, e.g. by as much as 25% to 30% in GSM and EDGE applications.

We propose a receiver structure that jointly estimates the channel, equalizes the transmitted symbols, and decodes the data by expanding the turbo equalization structure to include a separate soft-input channel estimator, based on a Kalman state estimator. This channel estiamtor performs well at relatively low signal to noise ratio (SNR) without training. By assuming an initial nominal channel estimate, the data is first equalized, potentially with significant errors. After decoding, the soft outputs from the SISO decoder are then used for channel esitmation, together with the received signal from the channel. The algorithm could similarly be bootstrapped with one of a variety of blind equalization algorithms.

It has been observed [4] that the priors of the first iteration play a crucial role in turbo equalization performance. If the priors from the previous SISO device are not sufficiently accurate, say due to channel estimation error, subsequent iterations may not substantially improve bit error rate (BER) performance. In the algorithm proposed in this paper, initially inaccurate priors due to a poor channel estimate are discarded in the first few iterations to prevent error propagation and to bootstrap the channel estimator. By doing this, the first few stages are dedicated to channel estimation without affecting the subsequent equalization and decoding procedure. After a few iterations, using information from the whole data frame, the channel estimate becomes more accurate and eventually more accurate than could have been obtained from training data alone with standard methods for channel estimation.

4. SOFT INPUT KALMAN CHANNEL ESTIMATOR

In this section, a soft input channel estimator is presented based on a Kalman estimator structure. For BPSK signaling (± 1), the log likelihood ratio (LLR) of the data can be converted to priors $p_m(n) = p(b(n) = b_m | r_1^K)$, $b_m \in \{-1, 1\}$

using the following relation,

$$p_0(n) = \frac{1}{1+\exp(\Gamma^p_{dec}(b(n)))} \quad (2)$$

$$p_1(n) = \frac{\exp(\Gamma^p_{dec}(b(n)))}{1+exp(\Gamma^p_{dec}(b(n)))} \quad (3)$$

where $\Gamma^p(b(n))$ is the interleaved version of $\Gamma^p_{dec}(a(n))$. The mean value and variance of each received symbol at each sampling time are expressed as $\bar{b}(n) = \sum_{m=0}^{1} b_m p_m(n)$, and, $\sigma_{\tilde{b}}^2(n) = \sum_{m=0}^{M-1} |b_m - \bar{b}(n)|^2 p_m(n)$ respectively. Again, BPSK signaling allows the simple expression of $\sigma_{\tilde{b}}^2(n)$ as $\sigma_{\tilde{b}}^2(n) = 1 - \bar{b}^2(n)$. In order to exploit the soft output of the turbo equalizer, we treat the transmitted signal as a stochastic signal, rather than a deterministic one, such that the transmitted signal can be decomposed into two parts, as $b(n) = \bar{b}(n) + \tilde{b}(n)$. As stated previously, $b(n)$ is modeled as a random process with zero mean and uncorrelated samples, through the assumption of the efficient interleaving. Correspondingly, it is given that $E[\tilde{b}(n+k)\tilde{b}^*(n)] = \sigma_{\tilde{b}}^2(n)\delta(k)$. With the above taken into consideration, the channel estimation formulation is as follows,

$$\mathbf{b}(n) = \bar{\mathbf{b}}(n) + \tilde{\mathbf{b}}(n), \quad (4)$$

$$\mathbf{h}(n+1) = \mathbf{F}\mathbf{h}(n) + \mathbf{G}\mathbf{v}(n), \quad (5)$$

$$r(n) = \mathbf{h}^H(n)\bar{\mathbf{b}}(n) + g(n), \quad (6)$$

where $\mathbf{h}(n) = (h_{n,0}, \cdots, h_{n,L-1})^H$, $\mathbf{b}(n) = (b(n), \cdots, b(n-L+1))^T$, $\bar{\mathbf{b}}(n) = (\bar{b}(n), \cdots, \bar{b}(n-L+1))^T$, and $\tilde{\mathbf{b}}(n) = (\tilde{b}(n), \cdots, \tilde{b}(n-L+1))^T$. Additionally, it is assumed that $E[\mathbf{v}(n+k)\mathbf{v}^H(n)] = \mathbf{Q}_v\delta(k)$, $E[w_0(n+k)w_0(n)] = \frac{N_0}{2}\delta(k)$. And $g(n) = \mathbf{h}^H(n)\tilde{\mathbf{b}}(n) + w_0(n)$ is introduced to incorporate the soft information in the Kalman filter structure. In (6), the observation equation is expressed in terms of $\bar{\mathbf{b}}(n)$ replacing the hard decision, $\mathbf{b}(n)$, and the uncertain component $\tilde{\mathbf{b}}(n)$ in the data is absorbed into the newly defined noise term $g(n)$. The auto-correlation of $g(n)$ is found as

$$E[g(n+k)g^*(n)] = tr\{\mathbf{F}^k\mathbf{R}_h(n)\mathbf{K}_{\tilde{b}}(n+k,n)\} + \frac{N_0}{2}\delta(k), \quad (7)$$

where $tr(\cdot)$ is the trace, and $\mathbf{R}_h(n) = E[\mathbf{h}(n)\mathbf{h}^H(n)]$, and $\mathbf{K}_{\tilde{b}}(n+k,n) = E[\tilde{\mathbf{b}}(n+k)\tilde{\mathbf{b}}^H(n)]$ with entries

$$[\mathbf{K}_{\tilde{b}}(n+k,n)]_{i,j} = \begin{cases} \sigma_{\tilde{b}}^2(n-k-i+1) & \text{if } i-j=k \\ 0 & \text{otherwise} \end{cases} \quad (8)$$

where $1 \le i,j \le L$. Since $g(n)$ is neither white nor Gaussian in general, the Kalman filter is not an optimal minimum mean square error (MMSE) estimator. However, in case that $g(n)$ is white, the Kalman filter is known to be the best linear unbiased estimator. We can show that $g(n)$ is uncorrelated based on the wide sense stationary uncorrelated scattering (WSSUS) multi-path fading channel model. Then, the transition matrix $\mathbf{F}$ can be represented as a diagonal matrix, as can $\mathbf{R}_h(n+k,n)$. With the aid of these two additional conditions and (8), it can be readily shown that $tr\{\mathbf{F}^k\mathbf{R}_h(n)\mathbf{K}_{\tilde{b}}(n+k,n)\} = 0$. Correspondingly, (7) can be simplified as

$$E[g(n+k)g^*(n)] = tr\{\mathbf{Q}_g(n)\}\delta(k) \quad (9)$$

where $\mathbf{Q}_g(n) = \mathbf{R}_h(n)\mathbf{K}_{\tilde{b}}(n,n) + \frac{N_0}{2}\delta(k)$. Therefore, the following recursion produces the best linear channel estimate, in a minimum mean square error sense,

$$\hat{\mathbf{h}}(n|n-1) = \mathbf{F}\hat{\mathbf{h}}(n|n), \quad (10)$$

$$e(n|n-1) = r(n) - \hat{\mathbf{h}}^H(n|n-1)\bar{\mathbf{b}}(n), \quad (11)$$

$$\mathbf{k}(n) = \frac{\mathbf{P}(n|n-1)\mathbf{b}(n)}{tr\{\mathbf{Q}_g(n)\} + \bar{\mathbf{b}}^H(n)\mathbf{P}(n|n-1)\bar{\mathbf{b}}(n)}, \quad (12)$$

$$\hat{\mathbf{h}}(n|n) = \hat{\mathbf{h}}(n|n-1) + \mathbf{k}(n)e(n|n-1)^*, \quad (13)$$

$$\mathbf{P}(n+1|n) = \mathbf{F}[\mathbf{I} - \mathbf{k}(n)\bar{\mathbf{b}}^H(n)]\mathbf{P}(n|n-1)\mathbf{F}^H + \mathbf{G}\mathbf{Q}_v(n)\mathbf{G}^H, \quad (14)$$

where $\mathbf{P}(n|n-1) = E[\epsilon(n|n-1)\epsilon^H(n|n-1)]$ with $\epsilon(n|n-1) = \mathbf{h}(n) - \hat{\mathbf{h}}(n|n-1)$, and $\mathbf{k}(n)$ is the Kalman gain vector. The undetermined term $\mathbf{Q}_g(n)$ can be recursively computed using the previous estimate $\hat{\mathbf{h}}(n|n-1)$ by

$$\mathbf{R}_h(n) = \hat{\mathbf{h}}(n|n-1)\hat{\mathbf{h}}^H(n|n-1) + \mathbf{P}(n|n-1) \quad (15)$$

And, $\mathbf{Q}_g(n)$ is found by

$$\mathbf{Q}_g(n) = diag[\sigma_{\tilde{b}}^2(n), \cdots, \sigma_{\tilde{b}}^2(n-L+1)]\mathbf{R}_h + \frac{N_0}{2}. \quad (16)$$

5. SIMULATIONS AND RESULTS

For the following simulations, the recursive systematic convolutional (RSC) code with generation polynomials, $\mathbf{g}_0 = [1,0,1]$, $\mathbf{g}_1 = [1,1,1]$, is used in an encoder, with $R_c = 1/2$ and $L_c = 3$. The channel used is time-invariant with impulse response $\mathbf{h} = [0.4803, 0.7339, 0.4803]$ which displays a strong null near $\omega = \pi$. At the receiver, the BCJR algorithm is used as a detector, and the LOGMAP algorithm is employed as a decoder. In the channel estimator, the channel is expressed as a first order auto-regressive random process of which state transition matrix, $\mathbf{F} = \lambda^{-1/2}\mathbf{I}_{L\times L}$.

We assume the dominant tap location and its sign are known by the initialization step, for example as determined from a blind bootstrap or data autocorrelation. According to this information, an implicit channel estimate is initialized as $\hat{\mathbf{h}} = [0,1,0]$. The information sequence of length, $L_a = 200$ is encoded using the given RSC code. The transmitted frame is composed of a training sequence of length L_p and coded data of length $L_s = 400$. A set of 500 simulations were run for each different SNR and training period. The

AR parameters are adjusted to provide a nearly stationary channel as $\lambda = 0.999$.

Fig. 3 depicts the BER performance of the proposed algorithm with $L_p = 0$ when the soft output of every iteration including the first iteration is used for subsequent iterations. The performance of the turbo equalizer with perfect channel knowledge and those of turbo equalizers equipped with the (hard decision) Kalman channel estimator for $L_p = 0$, $L_p = 40$ are also presented for a comparison. Since negligible performance improvement is observed after 5th iteration, the BER curves of the 1st and the 5th iteration were illustrated. The BER of the proposed receiver stays near 0.1 at the 1st iteration regardless of the input SNR due to the lack of an accurate channel estimate. As iterations continue, the performance of the proposed algorithm improves gradually, but it shows a large gap compared to the performance of the receiver with the perfect channel knowledge or that with a training sequence. In Fig. 4, the performance of the proposed algorithm is again shown against that of the conventional turbo equalizer, when only the first two iterations are devoted to channel estimation in the proposed method. Note that after two iterations of channel estimation using the proposed soft input estimator, followed by subsequent turbo equalization, the performance surpasses that of the trained turbo equalizer with $L_p = 40$ at approximately $6.5dB$ and approaches that of the turbo equalizer with perfect channel knowledge. Roughly a $1dB$ gain is observed as compared to the hard decision directed Kalman channel estimator.

6. CONCLUSIONS

This paper develops an algorithm for joint channel estimation, equalization and decoding when data protected by a convolutional code is sent over an unknown channel. Using a soft input Kalman channel estimator in conjuction with turbo equalization, the proposed receiver provided significant gains in BER performance without the need for a training data.

7. REFERENCES

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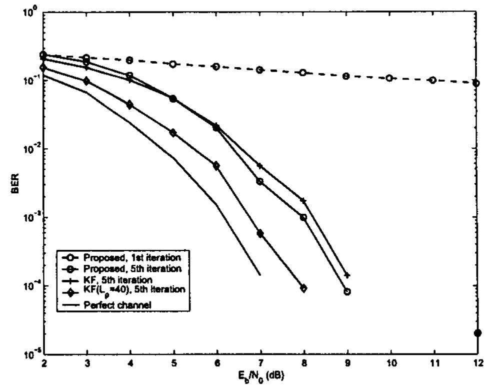

Fig. 3. Performance of the proposed receiver(KF: turbo equalizer with Kalman channel estimator)

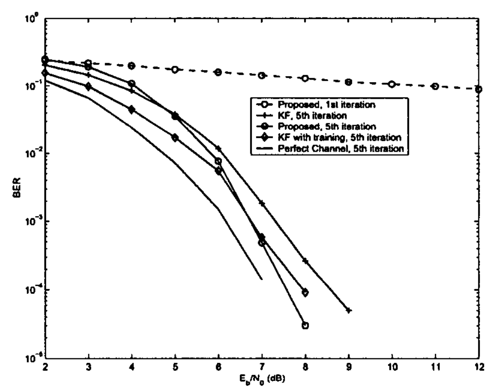

Fig. 4. Performance of the proposed receiver with 1st and 2nd iterations as the channel estimator (KF: turbo equalizer with Kalman channel estimator).

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