Linear Complexity Multiuser Detector using Joint Successive Interference Cancellation

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***Abstract*— We present a linear complexity multiuser detector that employs a joint successive interference cancellation (SIC) technique to achieve better performance than the standard SIC detector. The key feature of Joint Successive Interference Cancellation (JSIC) is that it considers two users jointly at each stage of successive decoding. For the two-user case, the JSIC detector gives the maximum likelihood (ML) solution. Simulations also show that for higher number of users, JSIC gives consistently better performance than the standard SIC detector, both in terms of error probability of a given user and the joint error probability for K users.**

I. Introduction

Successive or serial interference cancellation (SIC) [1] is an attractive low-complexity detection scheme for Code Division Multiple Access (CDMA) systems. However, the performance of the standard SIC detector falls off rapidly as the number of interfering users increases. Existing interference cancellation techniques also include parallel interference cancellation (PIC) [3], [4] which updates all user symbols simultaneously but it has the disadvantage of suffering from limit cycles [1]. Recently, a generalized version of SIC, in which more than one user's bit is updated at a time has been proposed [6]. It achieves the local maximum likelihood (LML) solution but the complexity is linear only with a neighborhood size of one, and grows as the neighborhood size increases. An alternative approach is Parallel Arbitrated Successive Interference Cancellation (PASIC) which employs low-complexity multi-stage detection [5]. PASIC performs a pruned maximum likelihood search over sets of bits decoded using successive interference cancellation.

We present a linear complexity joint successive interference cancellation (JSIC) scheme that achieves the ML solution for the two-user case and gives consistently better performance than the standard SIC detector for multiple users. The JSIC detector can be combined with the PASIC approach to give Parallel Arbitrated Joint Successive Interference Cancellation (PAJSIC) to achieve additional performance gains.

II. Maximum-Likelihood Decision Regions for a Two-User System

Consider the signal space spanned by two users with signatures $s_i(t)$ and $s_j(t)$ transmitting BPSK signals with bits b_i and b_j. The user signals $s_i(t)$ and $s_j(t)$ can be represented as vectors $\vec{s}_i$ and $\vec{s}_j$ respectively in signal space. Fig. 1 below shows the parallelogram $ABCD$ generated by the four possible signals $\{b_i\vec{s}_i + b_j\vec{s}_j : b_i, b_j \in (-1, +1)\}$. Note that

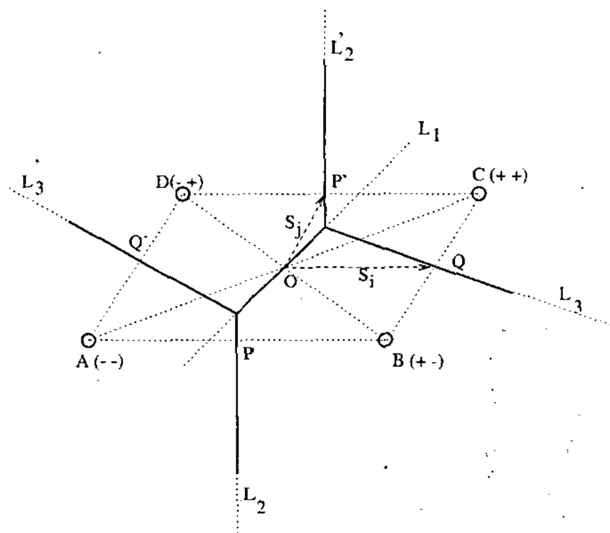

Fig. 1. ML Decision regions for two users

the boundaries of the ML decision regions of a particular constellation point are formed by the perpendicular bisectors of the lines joining itself with some, but not all, of the other three constellation points. This is true not only for a two-user constellation, but in general for K users.

Definition 2.1: The significant neighbor class (SNC) of a given constellation point P is the set of constellation points such that the set $\mathcal{L}(P)$ of perpendicular bisectors of the lines joining P to members of its significant neighbor class completely define the Maximum-Likelihood decision region of P.

As for example, in Fig. 1, the SNC of the point $(+,+)$ is $\{(-,+),(+,-)\}$ and $\mathcal{L}(+,+) = \{L_2', L_3\}$

Therefore, to determine whether or not a received signal point is closest to a candidate ML solution P, we need to only check whether it is completely enclosed by $\mathcal{L}(P)$, i.e., we only need to check against members of its significant

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neighbor class. From Fig. 1 we see that we see that the four ML decision regions of the two-user system are separated by the perpendicular bisector L_1 of the shorter diagonal BD and the bisectors of the four sides of the parallelogram. This is formally stated in the following theorem.

Theorem 2.1: For a two-user system, the four constellation points lie along the vertices of the parallelogram formed by the two user signals (refer Fig. 1). The significant neighbors of each vertex will be as stated below.

(i) *Any vertex on the longer diagonal will have only its adjacent vertices as significant neighbors.*

(ii) *Any vertex on the shorter diagonal of the parallelogram will have all the other three vertices as its significant neighbors.*

Before we prove Theorem 2.1, we need to prove some intermediate results. We will first state the following theorem without proof.

Theorem 2.2: For any parallelogram, the shorter diagonal faces an acute angle, and the longer diagonal faces an obtuse angle. If the parallelogram is a rectangle, all the angles are right angles and the diagonals are equal.

Now we will state and prove the following geometrical result.

Theorem 2.3: For any parallelogram $ABCD$, with BD as the shorter diagonal,

(i) *The vertex C lies outside the circumcircle of ΔABD*

(ii) *The vertex D lies inside the circumcircle of ΔABC*

In other words, the circumcircle of the triangle formed by the shorter diagonal and two adjacent sides can never enclose the whole parallelogram, while that formed with the longer diagonal and two adjacent sides always encloses it. When the parallelogram is a rectangle, all four vertices lie on the same circle.

Proof: Consider Fig. 2. Suppose the circumcircle S_1 of ΔABD does enclose the opposite vertex C'. By definition of the opposite vertex of a parallelogram, A and C' must lie on opposite sides of BD. We extend BC' so that it meets S_1 at the point X. From Fig. 2, we see that $\angle BAD + \angle BXD = 180°$ because they are opposite vertices of a cyclic quadrilateral.

$$\begin{aligned} \angle BC'D &= \angle C'DX + \angle DXC' \\ &= \angle C'DX + 180° - \angle BAD \\ &> 180° - \angle BAD \qquad (1) \end{aligned}$$

Also, $\angle BC'D = \angle BAD$, because A and C' are opposite vertices of a parallelogram. Therefore, putting this into Equation 1, we get $\angle BAD > 90°$. But, by Theorem 2.2, $\angle BAD$ faces the shorter diagonal BD and therefore, must be acute. Hence, we have a contradiction. This proves the first part of Theorem 2.3.

Now consider Fig. 3 below. We wish to show that the circumcircle S_2 of ΔABC, AC being the longer diagonal, always encloses the opposite vertex D. Suppose the opposite vertex is at D' outside S_2. AD' crosses S_2 at X. Again, since the exterior angle of a triangle is the sum of two

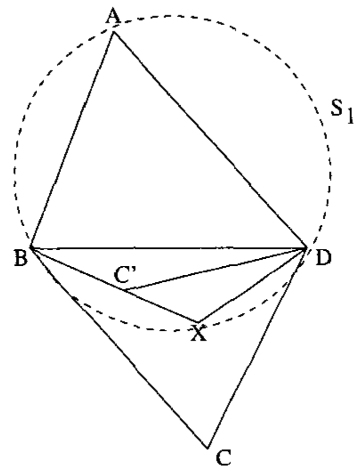

Fig. 2.

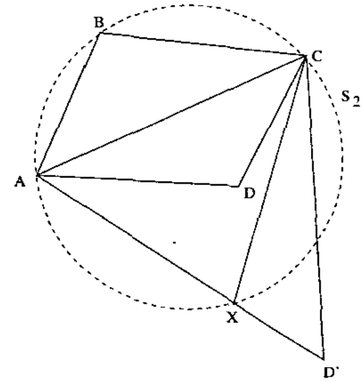

Fig. 3.

opposite interior angles, $\angle AXC > AD'C$. Also, since B and X are opposite vertices of the cyclic quadrilateral $ABCD$, $\angle ABC + \angle AXC = 180°$. Since D' and B are opposite vertices of parallelogram $ABCD$, $\angle ABC = \angle AD'C$. Therefore $180° - \angle ABC > \angle AD'C = \angle ABC$ which implies that $\angle ABC < 90°$. However, since AC is the longer diagonal, by Theorem 2.2, $\angle ABC$ must be obtuse. Hence, by contradiction the second part of Theorem 2.3 is proved.

Now we prove Theorem 2.1 as follows. Consider the two-user system shown in Fig. 4. The parallelogram $ABCD$ is generated by the four possible bit combinations of the two user signals $\vec{s}_1$ and $\vec{s}_2$. The four constellation points corresponding to $\{b_1\vec{s}_1 + b_2\vec{s}_2 : b_1, b_2 \in (-1, +1)\}$ form the vertices of the parallelogram. S_1 and S_2 are the circumcircles of ΔBCD and ΔBAD respectively. By Theorem 2.3, A and C lie outside S_1 and S_2 respectively. The perpendicular bisectors of BC and CD are L_3 and L_2 respectively and meet at the center S of S_1. Let us define $\Omega(L_2, L_3, C)$ as the region enclosed between L_2 and L_3 that includes the point C. Let X be the point at which AC crosses S_1. Let L_6 and L_7 be the perpendicular bisectors

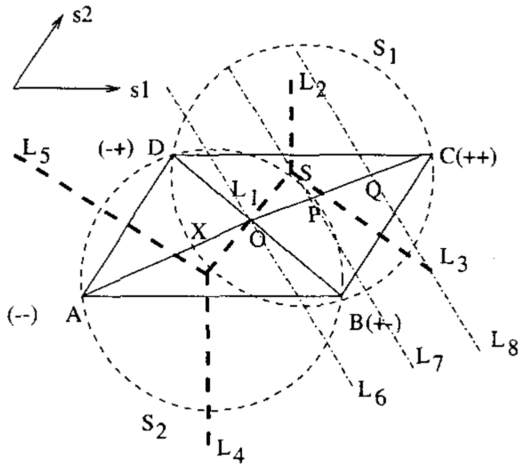

Fig. 4.

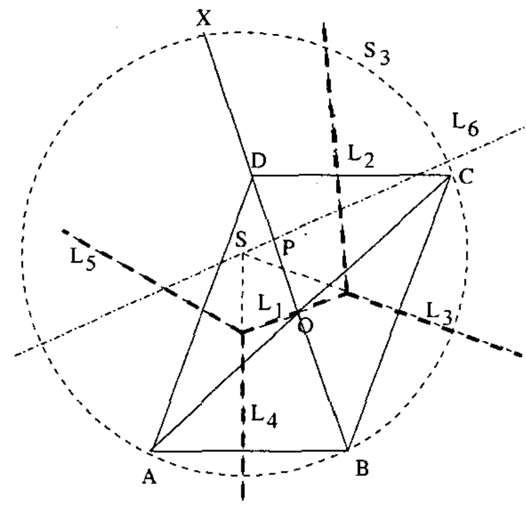

Fig. 5.

of AC and XC respectively. L_7 intersects AC at P. L_7 will pass through S since perpendicular bisectors of any chord of a circle passes through its center. L_7 will also be tangent to $\Omega(L_2, L_3, C)$ and parallel to L_6 since they are perpendiculars to the same straight line.

Consider any straight line L_8 parallel to L_7 and crossing $\Omega(L_2, L_3, C)$. From Fig. 4, we can see that L_8 intersects AC at Q and since L_7 is tangential to $\Omega(L_2, L_3, C)$, we must have $PC > QC$. Since A lies outside S_1, $AC > XC$. This implies $OC > PC$ (O and P are midpoints of AC and XC respectively). Therefore, L_6 does not cross $\Omega(L_2, L_3, C)$. Therefore, $\Omega(L_2, L_3, C)$ is the ML decision region of C and is completely determined by the perpendicular bisectors of DC and BC. Therefore, B and D are the only significant neighbors of C. Since the perpendicular bisector of AC never crosses $\Omega(L_2, L_3, C)$, A and C are not significant neighbors. By similar argument, B and D are the only significant neighbors of A. This proves the first part of Theorem 2.1.

However, consider Fig. 5. S_3 is the circumcircle of ΔABC with center at S and encloses the point D, since AC is the longer diagonal. Extend BD to meet S_3 at X. L_3 and L_4 bisect BC and AB respectively. The line L_6 is the perpendicular bisector of BX and intersects BX at P. Let $\Omega(L_3, L_4, B)$ is the region enclosed by the lines L_3 and L_4 and including B. The ML decision region of B will be either $\Omega(L_3, L_4, B)$ or a subset of it. Since the perpendicular bisectors of a chord of a circle meet at the center, L_6 is tangential to $\Omega(L_3, L_4, B)$ as is evident from the figure itself. Also, since O is the midpoint of BD and $BD < BX$, we must have $BO < BP$. Therefore, the line L_1 (perpendicular bisector of BD) crosses $\Omega(L_3, L_4, B)$. Therefore the ML decision region of B is given by the region enclosed by L_1, L_3, and L_4 and including the point B. Therefore, A, C and D are all significant neighbors of B. Similarly, A, B and C are all significant neighbors of D. This completes the proof of the second part of Theorem 2.1.

III. Two-User Maximum Likelihood (ML) Detector

In this section, we will describe a decision-driven detector that gives ML decisions for the two-user system. Fig. 1 is repeated below for convenience. If $\rho = < s_i(t), s_j(t) >$ [1] is the correlation between the user signals $s_i(t)$ and $s_j(t)$, OB represents the signal $p(t) = s_i(t) - \text{sgn}(\rho) s_j(t)$, which can be pre-computed. The ML detector estimates b_i from the received

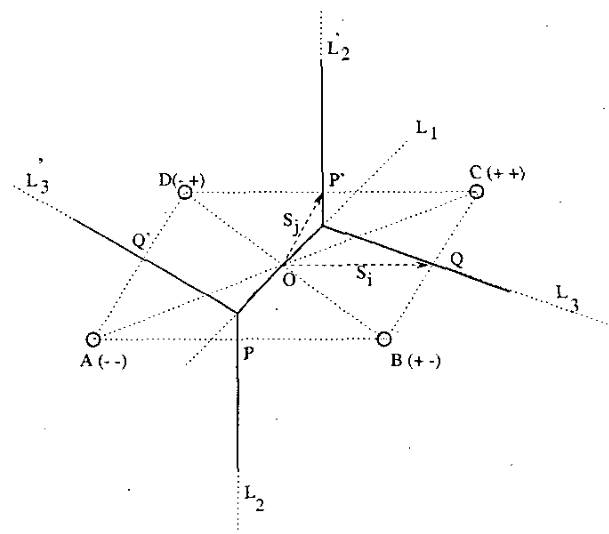

Fig. 6. ML Decision regions for two users

signal $y(t) = b_i s_i(t) + b_j s_j(t) + n(t)$, ($n(t)$ being the noise added by an AWGN channel) in two steps:

1) Decide to which side of L_1 the received point lies:

$$\hat{b} = \text{sgn}(< y(t), p(t) >) \qquad (2)$$

[1]wherever applicable, $< \cdot >$ denotes the standard inner product operation.

2) Move to P if $\hat{b} > 0$ (or P' if $\hat{b} < 0$) and decide on which side of the line L_2 (or L'_2) the received point lies:

$$\hat{b}_i = \text{sgn}(< (y(t) + \hat{b}s_j(t)), s_i(t) >) \quad (3)$$

Then b_j is estimated as:

$$\hat{b}_j = \text{sgn}(< (y(t) - \hat{b}_i s_i(t)), s_j(t) >) \quad (4)$$

Remark: Fig. 6 assumes $\rho > 0$. For the case $\rho < 0$, Equation 3 should be replaced by

$$\hat{b}_i = \text{sgn}(< (y(t) - \hat{b}s_j(t)), s_i(t) >) \quad (5)$$

IV. THE JSIC DETECTOR

Now consider a K-user CDMA system in which $\mathcal{S} = \{s_i\}_{i=1}^{K}$ is the ordered set of user signatures, arranged according to some appropriate criterion. A popular approach is to order the users in decreasing sequence of received powers. We could also scramble this arrangement and place the strongest user next to the weakest, then the second strongest user and the second weakest user, and so on. Let $\mathcal{B} = \{b_i\}_{i=1}^{K}$ be the transmitted bits over an AWGN channel.

The JSIC detector estimates $\mathcal{B}$ from the received signal $y(t)$ in $2(K-1)+1$ steps as follows:

1) Take the first two users and estimate b_1 from $y(t)$ using $s_1(t)$ and $s_2(t)$ in two steps using the two-user ML detector described in Section 2, ignoring users $3, 4, \ldots K$.
2) Subtract the effect of user 1 from $y(t)$ to generate

$$y_1(t) = y(t) - \hat{b}_1 s_1(t) \quad (6)$$

3) Take users 2 and 3, and estimate b_2 from $y_1(t)$ using $s_2(t)$ and $s_3(t)$ as in Step 1, again ignoring users $4, 5, \ldots, K$.
4) Continue in this fashion until we come to the last two users. Estimate b_{K-1} from $y_{K-2}(t)$ in two steps and estimate b_K in the last step of the ML detector (ref. Equation 4).

Remark: The key idea behind JSIC is to account for the closest interferer in an ordered set by using a two-user ML kernel. Thus we can come up with better estimates for each user at each stage of successive decoding. It is easy to verify that the JSIC detector performs $2(K-1)+1$ inner product operations to estimate $\mathcal{B}$, and therefore has linear complexity.

A. The PAJSIC detector

The PAJSIC detector performs P permutations on the user order and for each permutation computes the JSIC estimate. Then it computes the local maximum likelihood estimate over these P estimates of $\mathcal{B}$. PAJSIC is similar to PASIC [5] which employs a multi-stage SIC detector to compute the estimate for each permutation.

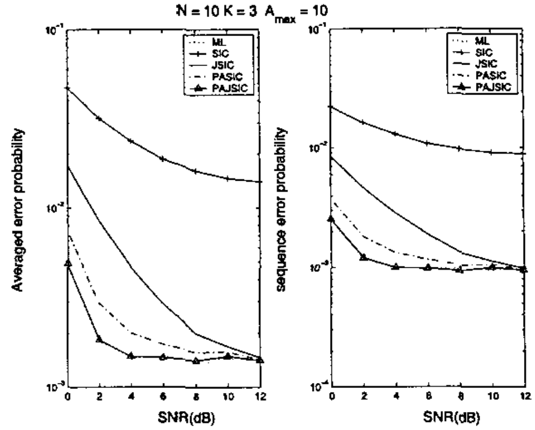

Fig. 7. Comparison of (a) averaged bit error probability and (b) sequence error probability between SIC, JSIC, PASIC and PAJSIC for three users

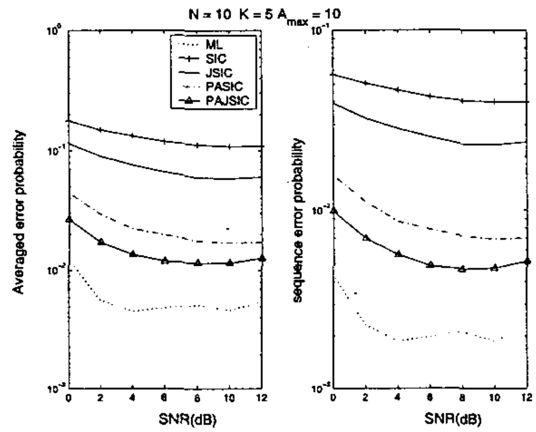

Fig. 8. Comparison of (a) averaged bit error probability and (b) sequence error probability between SIC, JSIC, PASIC and PAJSIC for five users

V. SIMULATION RESULTS

Simulations show that the JSIC detector consistently performs better than the standard SIC detector. All signature sequences were random binary sequences of length N. Fig. 7, Fig. 8 and Fig. 9 compare the averaged bit error probability and sequence error probability of the detectors for different numbers of users. A_{max} denotes the amplitude of the strongest user normalized against that of the weakest user. The averaged bit error probability is the single-user error probability averaged over the number of users. The sequence error probability is the probability of making an erroneous estimate of the K-bit transmitted sequence (sometimes called the joint error probability). The corresponding error probability of the ML detector is also shown as a lower bound.

We observe that though the performance margin between JSIC and SIC (as well as between PAJSIC and PASIC) decreases with increase of K, the JSIC (and PAJSIC) detectors consistently perform better than the corresponding SIC

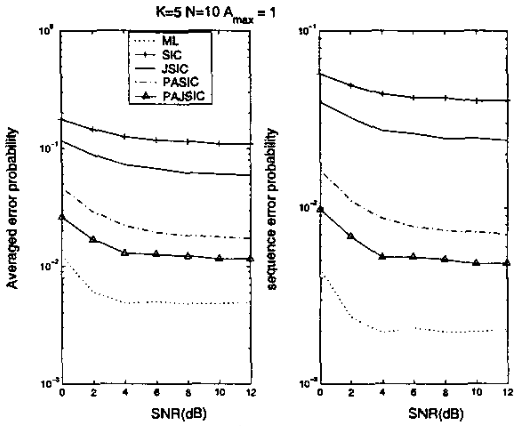

Fig. 9. Comparison of (a) averaged bit error probability and (b) sequence error probability between SIC, JSIC, PASIC and PAJSIC for five equal-energy users

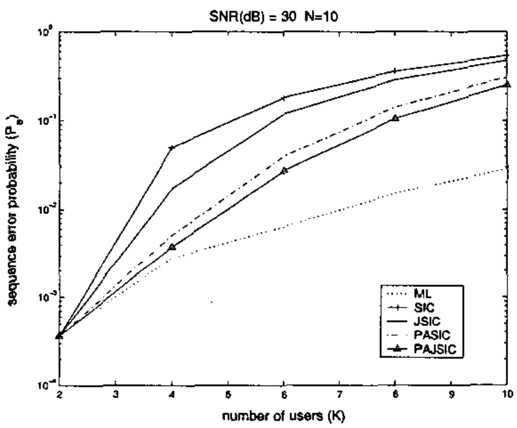

Fig. 10. Comparison of sequence error probability between SIC, JSIC, PASIC and PAJSIC for SNR=dB plotted against number of users

implementations.

Fig. 10 shows simulations in which the SNR is held fixed at 30 dB and the sequence error probabilities of the different detectors are observed over an increasing number of users. The JSIC (or PAJSIC) detectors again give consistently better performance over SIC (or PASIC).

VI. Conclusion

We have proposed a multi-user detector that employs successive interference cancellation taking into account the joint effect of the two closest users in an ordered set. For the two-user case, the JSIC detector will give the maximum likelihood solution and hence achieves optimum near-far resistance. However, the performance of the JSIC detector deviates from that of the ML detector as we increase the number of users. Simulations show that the JSIC detector consistently outperforms the standard SIC detector, and the PAJSIC detector yields performance gains over PASIC.

Acknowledgment

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