# MULTI-DIRECTIONAL DECISION FEEDBACK FOR 2D EQUALIZATION

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**ABSTRACT**

We propose an equalization algorithm that employs multiple decision-feedback equalizers (DFE)s operating in different directions and arbitration among the outputs of these equalizers to mitigate the effects of two-dimensional intersymbol interference (ISI). The multi-directional arbitrated DFE (MAD) exploits directional diversity to reduce the effects of error-propagation while maintaining complexity on the same order as a DFE. Simulation results show that, when four DFEs are used, the MAD algorithm can achieve substantial gains over a single DFE, including gains of over 10 dB at $10^{-2}$ BER for simulations in this paper.

## 1. INTRODUCTION

Like their one-dimensional (1D) counterparts, systems that suffer from two-dimensional (2D) intersymbol interferece (ISI) require equalization to mitigate the effects of ISI. Much of the previous research in 2D equalization studies extensions of 1D algorithms. Several of these extensions approach the problem of 2D equalization as two independent 1D problems, performing equalization first in one direction and then in the other [1, 2]. If the 2D ISI channel is separable, i.e. can be written as a 1D convolution in one dimension followed by a 1D convolution in the other dimension, then such an approach may be reasonable. For channels that are not separable, however, such methods ignore a subset of the ISI present and will likely suffer performance degradation with respect to 2D linear methods as a result. They will always be inferior to methods that do not process the data separably. For example, the maximum likelihood solution cannot be performed in a separable manner even when the channel is separable.

Extensions of linear minimum mean square error (MMSE) equalization to 2D channels [3, 4], methods based on the Viterbi algorithm [5, 6], as well as iterative methods for joint equalization and decoding [7] and for equalization alone [8] have all been applied to the 2D channel. The 2D equalization algorithms closest to the work in this paper are those that incorporate elements of the decision feedback equalizer (DFE). The standard DFE employs two filters to mitigate the effects of ISI. The feedforward filter accepts received data from the channel as input and attempts to cancel precursor (anticausal) ISI. The feedback filter receives hard estimates of previous symbols as input and attempts to remove residual postcursor (strictly causal) ISI. Heanue, et al. [9] propose using the Viterbi algorithm along rows of the received array and estimates of symbols in previous rows to subtract their contributions. Neifeld, et al. [10] propose an extension of the 2D MMSE equalizer based on parallel iterative DFEs.

We present an algorithm that employs DFEs operating on multiple paths through the array of received data (scans). The algorithm then arbitrates among the outputs of these DFEs. This technique, which we refer to as the MAD (multi-directional arbitrated DFE) algorithm, is an extension of the bi-directional arbitrated DFE (BAD), which was developed for low-complexity equalization of 1D ISI channels [11, 12]. Decision feedback equalization suffers from error propagation. By performing decision feedback equalization in two directions and arbitrating between the resulting estimates, the BAD algorithm exploits time diversity and hence decorrelated error propagation effects to reduce these detrimental effects. The same benefits apply when such an approach is used for 2D equalization. In fact, because of its array structure, the 2D problem allows for a larger variety of equalization directions; while the DFE is essentially restricted to traveling only forward or backward through the vector of received data from a 1D channel, the DFE can follow any number of scans through a 2D array.

The remainder of the paper is organized as follows. We first present our system model. We then describe the three stages of the MAD algorithm: multi-directional processing, reconstruction, and arbitration. We present its performance on representative channels and explore the performance improvement achieved by arbitrating among the out-

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puts of four DFEs rather than two. We also examine the peformance of the MAD algorithm when applied to optical storage.

## 2. SYSTEM MODEL

Consider linear modulation over a real baseband, discrete-time, symbol-spaced two-dimensional channel $\{h[i,j]\}$, where $-u \leq i \leq d$ and $-l \leq j \leq r$. Data is transmitted in $N \times M$ arrays, $\{b[n,m]\}$, and the channel is corrupted by additive white Gaussian noise (AWGN). The channel output at index $[n,m]$ is given by

$$y[n,m] = \sum_{i=-u}^{d} \sum_{j=-l}^{r} h[i,j]b[n-i,m-j] + w[n,m],$$

where $\{w[n,m]\}$ are i.i.d. AWGN samples with variance $\sigma^2 = N_0/2$. It is assumed that the noise variance and the channel response are known to the receiver. In the simulations and discussions presented in this paper, we consider a binary phase shift keying ($b[n,m] \in \{-1,1\}$) modulation scheme. However, the MAD algorithm can be easily extended to larger symbol alphabets.

## 3. THE MAD ALGORITHM

Like the original 1D algorithm, the MAD algorithm consists of three stages: multi-directional processing, reconstruction, and arbitration. In the multi-directional processing stage, the array of received data is passed through multiple DFEs, each moving along a different scan. Though the structure of the data allows flexibility in choosing many paths, it also complicates the design of the DFE filters, since the ISI to be cancelled results from a 2D region of symbols around the symbol to be estimated, not just those to the right and left, and the notion of which interfering symbols are precursor and which are postcursor is dictated by the scan direction. Note that the samples and symbol estimates fed to the feedforward and feedback filters will not lie along a single row or column of the array. Rather, they will be drawn from a 2D region surrounding the symbol to be estimated.

For our simulation and discussion, we employ four DFEs. The first DFE moves from left to right along each row, beginning with the top row of the array and ending with the bottom. Hence, estimates of the symbols in rows above the current row and in the current row to the left of the current symbol have been generated and can be passed to the feedback filter. Symbols to the right of the current symbol in the same row and anywhere in the rows below have not yet been estimated, and hence their contributions to the current symbol are considered precursor ISI. The second DFE also operates along rows of the array, but it moves from right to left along each row and starts with the bottom row and ends with the top. Hence, symbols that contributed postcursor ISI when viewed by the first DFE contribute precursor ISI when viewed by the second DFE. Error propagation thus occurs in different directions in the two DFEs and can be reduced via arbitration. The third and fourth DFEs move along columns of the received array; the third moves from the top to the bottom of each column, starting with the leftmost column and moving to the rightmost. The fourth moves from the bottom to the top of each column, starting at the right and working to the left.

Though there are many possible scans, the four paths described above have two advantages. First, because any particular DFE is always moving in the same direction along rows or columns, the elements of the 2D channel that appear as causal and anticausal are the same for each symbol; hence, the DFE filter taps are invariant throughout the array (ignoring edge cases). If we chose a path that reversed direction at the end of each row, for example, the filter taps would no longer be constant (unless the channel were symmetric). Second, the paths of the second, third, and fourth equalizers can be viewed as equivalent to that of the first by a combination of transposing the array and flipping its elements column-wise and row-wise. Hence, the same procedure can be used to design the filter coefficients for all four DFEs simply by making the appropriate transposes and flips to the channel matrix when the filter taps are computed and to the received data when the DFE is implemented.

In the reconstruction stage of the MAD algorithm, the array of symbol estimates generated by each DFE is convolved with the channel to produce a noise-free estimate of the received array of samples. These four estimates of the received array serve as candidates in the arbitration process, which is the final stage. If the estimates of a particular symbol $b[n,m]$ generated by the four DFEs do not agree, arbitration is employed to determine the final output. Since the noise is Gaussian, the MAD algorithm uses Euclidean distance as a metric to choose among the candidate symbol estimates. The Euclidean distance between each noise-free estimate of the received array and the actual received array is computed over a window around the bit to be estimated as

$$\gamma_k[n,m] = \sum_{i=-W_1}^{W_2} \sum_{j=-W_3}^{W_4} |r[n+i,m+j]-\hat{r}_k[n+i,m+j]|^2$$

for $k = 1,2,3,4$, where $W1$, $W2$, $W3$, and $W4$ define the window. The final estimate of $b[n,m]$ is taken from the DFE for which $\gamma_k[n,m]$ is smallest.

## 4. RESULTS AND DISCUSSION

We first consider the performance of the MAD algorithm for optical storage. The optical storage channel (or point-spread function) is typically modeled as a symmetric 2D

Gaussian. We consider such a Gaussian channel with variance $\sigma_b^2 = 0.55$ and approximate it by a $3 \times 3$ discrete-time channel, normalized to have unit energy. For each DFE, we use a feedforward filter of length 6 (2 causal taps and 4 anti-causal taps) and a strictly causal feedback filter of length 4. Consider the first DFE, which operates from left to right on each row, starting at the top of the received array. For the estimation of symbol $b[n, m]$, the feedforward filter takes as input the vector of received samples

$$\left[y_{n,m-1} \;\; y_{n,m} \;\; y_{n,m+1} \;\; y_{n+1,m-1} \;\; y_{n+1,m} \;\; y_{n+1,m+1}\right].$$

The feedback filter takes as input the vector of symbol estimates

$$\left[\hat{b}_{n,m-1} \;\; \hat{b}_{n-1,m+1} \;\; \hat{b}_{n-1,m} \;\; \hat{b}_{n-1,m-1}\right].$$

The inputs to the feedforward and feedback filters of the other three DFEs are chosen similarly with respect to their scans. Any inputs to the filters that lie outside the received data and symbol estimate arrays are assumed to be 0. For arbitration, we use a window of size $3 \times 3$ centered on the symbol to be estimated.

One hundred arrays of size $100 \times 100$ were were passed through the channel and processed by the MAD receiver. Figure 1 shows the resulting bit error rate (BER) for $E_b/N_0$ between 5 and 25 dB. In order to explore the benefit gained by using four arbitration candidates, we present the results of arbitration between the outputs of only the two row-wise DFEs and only the two column-wise DFEs, as well as among all four DFE outputs. In addition, we plot the BER at the output of each individual DFE prior to any arbitration.

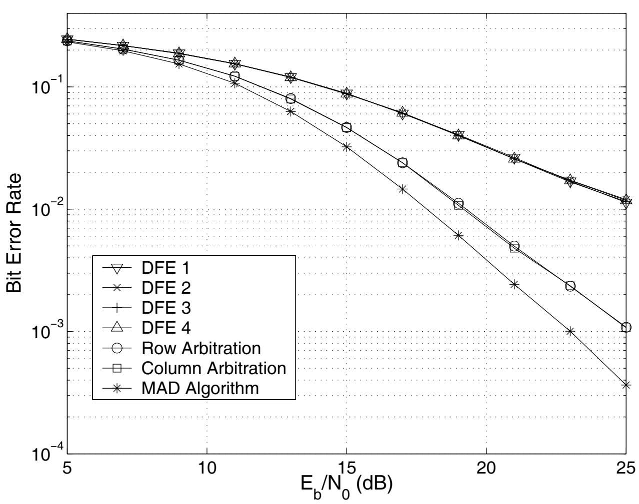


**Fig. 1**. Performance of the MAD algorithm on the optical storage channel. The four standard DFEs give identical results, as do the row-wise and the column-wise arbitration.

Because the channel is symmetric, the four DFEs operating in different directions see the same channel (i.e. the same channel elements correspond to pre- and post-cursor ISI). As would be expected, the BER achieved by the four DFEs is identical for this channel. Arbitration between only the two row-wise DFEs or only the two column-wise DFEs yields a gain of 1.5 to 6 dB. Arbitration among all four candidate sequences, however, yields a gain of up to 7 dB over a single DFE and up to 2 dB over arbitration between only two candidates, confirming the benefits of incorporating more arbitration candidates.

Another parameter of interest in the MAD algorithm is the size of the arbitration window. Figure 2 shows the BER achieved by the MAD algorithm (arbitrating among all four DFE outputs) on the optical storage channel for windows of size $1 \times 1$, $3 \times 3$, $5 \times 5$, and $11 \times 11$. The results reveal that the performance of MAD improves significantly (up to 4 dB) as the window size increases to $3 \times 3$ and remains essentially constant as the window size increases to $5 \times 5$. The performance degrades somewhat (up to 2 dB), however, for the largest window size considered. This likely indicates that, as the window grows beyond a certain size, the amount of additive noise included in the distance metric increases significantly, but information about the transmitted symbol of interest does not. Hence, selecting an appropriate arbitration window size is important for achieving the best possible performance from the MAD algorithm.

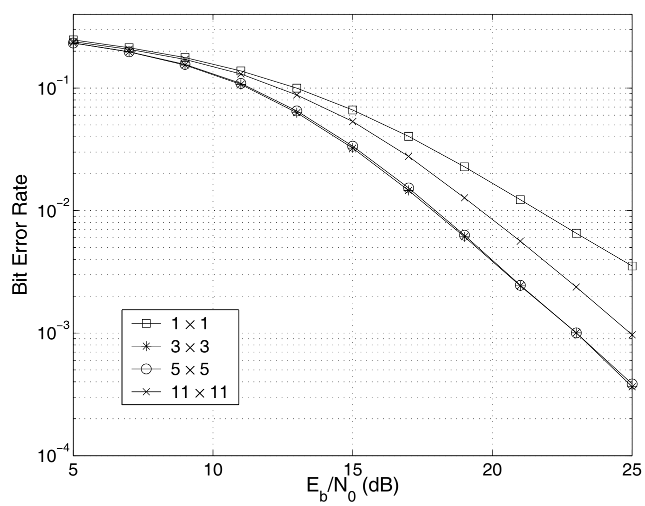


**Fig. 2**. Effect of arbitration window size on performance of the MAD algorithm. The $3 \times 3$ and $5 \times 5$ windows give nearly identical performance.

We also consider the performance of the MAD receiver on an asymmetric channel given by

$$\mathbf{h} = \begin{bmatrix} -.1599 & .1064 & .4397 \\ -.4239 & -.6158 & -.0139 \\ .0463 & .4403 & .1209 \end{bmatrix}.$$

All other parameters are the same as those used to generate the results in Figure 1. When the channel is asymmetric, it appears differently to each of the DFEs. The elements of $\mathbf{h}$ that contribute postcursor ISI in the view of the first DFE, for example, will contribute precursor ISI in the view of the second DFE. Hence, as Figure 3 shows, the four DFEs now give somewhat different BER values. When arbitration occurs between the outputs of only two DFEs (both row-wise DFEs or both column-wise DFEs), a gain of 1.5 to 9 dB over the best-performing single DFE is achieved. When arbitration among all four DFE outputs is performed, an additional gain of 1.5 to 7 dB can been seen, yielding overall gains of up to 13 dB. These results indicate that, by allowing for the generation of more than two arbitration candidates, the 2D extension of the BAD algorithm yields significant performance gains even beyond its 1D counterpart.

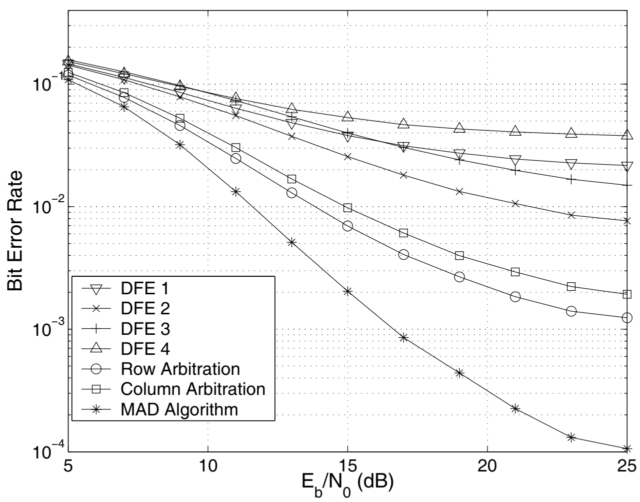


**Fig. 3**. Performance of the MAD Algorithm on an asymmetric channel.

## 5. CONCLUSION

We have presented an algorithm for 2D equalization that employs multi-directional DFEs. Based on the 1D BAD algorithm, the MAD algorithm processes the data array in several directions and arbitrates among the resulting bit estimates, using Euclidean distance as a metric. Simulation results show that the inclusion of four candidates for arbitration, allowed by the 2D channel and data structure, yields significant performance gains above both a standard DFE and an arbitration algorithm using only two candidates, especially for the asymmetric channel. The MAD algorithm maintains complexity on the same order as that of the standard DFE while yielding gains of up to 13 dB over the conventional structure, thus making it an attractive receiver choice for systems requiring low-complexity ISI mitigation.

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