Evolution of Prior Information in SISO Equalization

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Abstract — **Soft-input soft-output (SISO) devices play an important role in a variety of algorithms for communications problems. In an iterative structure, soft information is passed between a number of SISO devices, and the ensemble of the soft information evolves with each iteration. We develop and present a framework for analyzing the evolution of this soft information in a class of SISO linear equalizers.**

I. INTRODUCTION

Soft-input soft-output (SISO) devices are commonly used in communications algorithms for tasks such as detection, equalization, and decoding of received signals. In SISO devices, soft information about the transmitted symbols is passed to the device, and a new set of soft information is generated by the device. In an iterative scheme, this soft information evolves as the algorithm iterates and allows the algorithm to achieve performance gains over algorithms that use only hard outputs.

To understand these iterative SISO algorithms and their performance, we study the evolution of soft information in algorithms that use SISO MMSE equalizers; we treat the information as a random variable and study the effect of the SISO device as a transformation of that random variable. As a special case, we examine the evolution of the soft information in the LITE algorithm [3], an iterative SISO equalization structure.

II. PROBLEM DESCRIPTION

The input to a SISO device in a communications scenario is generally a priori probabilities (priors) on the transmitted symbols, and the output generated by the device is a new set of priors on the data. Because the outputs of the SISO device are stochastic quantities, we assume that the priors are independent samples drawn from a single distribution. We can then view the SISO device as a mapping from one distribution to another.

To reduce complexity, we quantize the possibly continuous probability density function (PDF) over the priors to a discrete probability mass function (PMF) and consider the transformation of this discrete random variable. The SISO device transforms this PMF to a possibly continuous distribution over the output priors, which is also quantized to the points of the original PMF. To analytically determine the output PMF of a SISO device for a particular input PMF, we consider the sample space composed of all possible inputs to the device. We transform each sample point individually and quantize the resulting PDFs to PMFs. To compute an overall output PMF for the SISO device, we weight each PMF by the probability of the sample point that generated it and sum the results.

To allow for a general framework, we assume that, conditioned on a given input, the output distribution of the SISO device is Gaussian. Thus, the weight at any particular point x_i of the output PMF can be calculated as

$$Pr(x_i) = \sum_{j=1}^{S} Pr(j) \left(\Phi\left(\frac{a-\mu_j}{\sigma_j}\right) - \Phi\left(\frac{b-\mu_j}{\sigma_j}\right) \right),$$

where S is the total number of points in the sample space, μ_j and σ_j^2 are the mean and variance of the device output conditioned on input j, and a and b are the upper and lower bounds of the region of the Gaussian distribution that is mapped to point x_i of the discrete PMF during quantization.

III. ANALYSIS OF THE LITE ALGORITHM

In order to demonstrate this analysis framework for MMSE equalizers, we examine the LITE algorithm, which iteratively passes soft information between two linear SISO MMSE equalizers. In the LITE algorithm, two channels are available, and data is interleaved prior to transmission over one of the channels. An MMSE linear equalizer is used at the output of each channel, and priors are passed between the two equalizers until a convergence criterion is reached.

In the LITE algorithm, the sample space consists of all possible combinations of transmitted symbols $\vec{b}$ and the priors $\vec{\epsilon}$ on those symbols. Because the MMSE equalizer output is a function of multiple received values from the channel, these inputs are vector rather than scalar quantities. Each sample point is transformed to an output PMF, and these PMFs are weighted and summed, as described above.

The LITE algorithm has been analyzed for the case in which all ones were transmitted and the case in which all possible sequences were transmitted. A five-point PMF with points equally spaced between 0 and 1 was used in the analysis. In both cases, the output of the analysis reveals that the PMF over the priors converges quickly, and is in fact nearly stationary after only two iterations. In addition, probability of error values predicted by the analysis closely match those generated by simulation, although the analysis gives an optimistic prediction due to the assumption of independent information in each iteration.

REFERENCES

[1] C. Berrou, A. Glavieux, and P. Thitimajshima, "Near Shannon Limit Error-Correcting Coding and Decoding: Turbo-Codes," *Proceedings of the IEEE International Conference on Communications*, pp. 1064-1070, May, 1993.

[2] T. Richardson, A. Shokrollahi, and R. Urbanke, "Design of Provably Good Low-Density Parity Check Codes", Submitted to *IEEE Transactions on Information Theory*, April, 1999.

[3] A. Singer, J. Nelson, and R. Koetter, "Linear Iterative Turbo-Equalization (LITE) for Dual Channels," *Proceedings of the Thirty-third Asilomar Conference on Signals, Systems, and Computers*, vol. 2, pp. 1670–1674, October, 1999.

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