

# A DFE COEFFICIENT PLACEMENT ALGORITHM FOR UNDERWATER DIGITAL ACOUSTIC COMMUNICATIONS

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## Abstract

We describe and evaluate an algorithm for determining decision-feedback equalizer (DFE) coefficient placements from an estimated channel response. Efficient tap placement is desirable to minimize equalizer computation in multichannel, underwater acoustic environments due to long, potentially sparse channel responses lasting hundreds of symbols or more. Although our algorithm relies on heuristics to simplify the placement search, it is based on taking advantage of the interaction between the minimum mean-squared error (MMSE) optimal feedforward and feedback filters, rather than optimizing over each separately. Tests of the automated routine on two diverse sets of data gathered from an at-sea experiment of an underwater acoustic communication system demonstrate the algorithm's ability to provide complete automation of the tap placement process while maintaining performance.

## I. Introduction

The underwater acoustic channel remains one of the most challenging environments for digital communications. Channel responses may last 200 ms or longer, and tend to vary considerably over the several seconds of a typical data packet. Spatial diversity combining is often necessary to combat high noise levels. Despite these difficulties, recent advances in underwater acoustic communication using adaptive equalization and phase compensation [1] have shown the viability of coherent communication techniques in these adverse conditions.

Decision-feedback equalizers (DFEs) have been used in conjunction with training data to equalize these types of channels. One critical problem in a successful implementation is to determine the support region of

the feedforward and feedback filters. Since their effects on the resulting DFE are highly coupled, the feedforward and feedback tap placements should be jointly determined.

A filter with too few taps will be unable to sufficiently compensate for the channel. Very long filters will be computationally prohibitive, unresponsive to channel fluctuations, and may require excessive training data to initially learn the equalizer coefficients. Therefore, the challenge is to develop an algorithm that can efficiently place feedforward and feedback taps only where necessary.

One of the biggest challenges in developing an automated tap placement algorithm is achieving robustness with respect to the diverse class of potential channel responses, as shown in Figs. 1 and 2 for several measured equivalent baseband responses. While some channel responses contain several resolvable multipath arrivals, others consist of an unresolvable block of arrivals which may require several hundred filter taps in the equalizer. Relatively simpler channels contain a well resolved main arrival, with little additional energy in the response.

## II. Sparse Decision-Feedback Equalizer

A DFE can be thought of as equalizing a channel in two steps: first, a feedforward section (linear filter) shapes the overall response appropriately, and then the feedback of sliced (quantized) outputs cancels postcursor intersymbol interference (ISI). We can express the output of a (nonsparsely) DFE as

$$\hat{d}[n] = \sum_{k=-L_f}^{L_f} a[k]r[n-k] + \sum_{k=1}^M b[k]\tilde{d}[n-k] \quad (1)$$

in which  $r[n]$  represents the baseband received data,  $\tilde{d}[n]$  the pre-sliced decision variables, and  $\hat{d}[n]$  the sliced (final) decision variables. For a multi-input experiment, e.g. a receive array, there will be one feedforward filter  $a_i[n]$  for each array element  $i$  and, for a

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fractionally-spaced equalizer, the received data  $r[n]$  will arrive at twice the rate of the decisions  $\tilde{d}[n]$  and  $\hat{d}[n]$ . Given a channel impulse response  $h[n]$ , symbol variance  $\sigma_d^2$  additive noise variance  $N_0$ , and a desired filter tap support (number of parameters), the minimum mean-squared error (MMSE) optimal DFE coefficients can be computed by

$$\begin{bmatrix} a[-L_1] \dots a[L_2] \end{bmatrix}^T = \mathbf{R}^{-1} \mathbf{p} \\ b[n] = h[n] * a[n], n = 1, \dots, M \quad (2)$$

where

$$\mathbf{p} = \sigma_d^2 \begin{bmatrix} h^*[L_1] \dots h^*[-L_2] \end{bmatrix} \quad (3)$$

and, when the additive noise is white, the matrix  $\mathbf{R}$  is given by

$$R_{ij} = \sigma_d^2 \sum_{k \in S} h^*[k-i] h[k-j] + N_0 \delta_{ij} \quad (4)$$

where  $S$  is the set  $[L_1 + 2, L_1 + M + 1]$ . The "estimated mean-square error" (EMSE) which would arise from the response estimate is given by

$$E \left[ (d[n] - \hat{d}[n])^2 \right] = \sigma_d^2 - \mathbf{p}^H \mathbf{R}^{-1} \mathbf{p} \quad (5)$$

where  $\mathbf{A}^H$  denotes the Hermitian transpose of  $\mathbf{A}$ . Extensions to fractionally-spaced and multi-element array systems are straightforward; see, for example, [2] for more details.

In (1),  $L = L_1 + L_2$  contiguous feedforward taps and  $M$  contiguous feedback taps are used. The goal of a sparse equalizer is to set as many of the values of  $a[n]$  and  $b[n]$  as possible to zero, while still correctly decoding the data or achieving some prescribed MSE. Existing methods [3,4] rely on thresholding an impulse response estimate for determining either the feedforward or feedback placements. However, the selection of a single threshold for a variety of channels becomes problematic for a robust, automated, multi-input routine. Further, direct reliance on the impulse response estimate ignores much of the interdependence of the MMSE-optimal feedforward and feedback filters.

Since their effects on the resulting DFE are highly coupled, the feedforward and feedback tap placements should be jointly determined. For example, if there are a large number of feedforward coefficients, a later multipath arrival may be nulled without the need of additional feedback. On the other hand, placing a small amount of feedback around the position of strong ISI may allow the feedforward filters to concentrate effort elsewhere and thus achieve a lower MSE. Unfortunately, an exhaustive search over all possible filter tap combinations is impractical; for example, for 6 element array data with maxima of 50 feedforward taps per array element and 100 feedback taps, there are a total of

$2^{400}$  distinct configurations. Methods based on the iterative procedure described in [5] also seem prohibitive for the number and span of potential taps.

The DFE structure presents considerable challenges to reducing the complexity of the placement search. One reasonable approach would be to use an exchange-type algorithm, which alternates between updating feedforward and feedback tap placements. Simple rules at each stage can then be geared toward optimizing more traditional, linear filters. We still consider the coupling between the two filters by using the initial feedback placement to optimize the number of feedforward taps, and vice-versa. In our final algorithm, we chose to use two such iterations, adding additional taps at each step to ensure that a performance criterion is met.

### III. Filter Tap Optimization Algorithm

Our goal is an algorithm that leads to successful equalization with a minimum number of computations. One might expect that this balance could be achieved by setting an MSE goal that ensures a minimum separation in the output constellation. Unfortunately, the EMSE from (5) does not correspond to the error in actual operation due to the imperfect nature of the channel estimate, unmodeled noise propagation which increases with filter length, and temporal changes in the channel response. Instead of looking at the absolute EMSE, we use a measure of model complexity akin to the MDL [6], roughly correlated with minimizing the sum of the EMSE and a term that is linear in the number of taps  $N$ ,

$$\gamma = E[\text{EMSE}] + \beta N \quad (6)$$

Note that if only feedforward or feedback coefficients are being added at each stage of the algorithm, (6) can be interpreted as a stopping criterion.

As stated earlier, some rules on tap placement are necessary to simplify the search. For the feedforward filter, we followed the approach in [4], restricting the support to a centralized, contiguous block (of the same size for each array element). Such filters attempt to concentrate energy from the main arrival, while steering ISI toward the positions of the feedback taps where it can be canceled. Placing sparse feedback taps was guided by the EMSE improvement of adding a single tap at any position to avoid calculating (2)-(5) for every possible configuration.

With these simplifications, our tap placement algorithm can be summarized by the following steps:

1. Ramp up: Add initial feedforward and feedback taps until  $\text{EMSE} = 0.5$ .

2. Feedback: Place additional feedback taps where they will improve EMSE by at least an amount  $\delta$ .
3. Feedforward: Increase  $L$ , the number of feedforward taps per array element, until a minimum is found for

$$\tilde{\gamma} = E[\text{MSE}] + \tilde{\beta}L. \quad (7)$$

4. Repeat Feedback step.

Fig. 3 demonstrates the algorithm on sparse, four element array data. The impulse response estimate from the first array element, available from a probe sent prior to the data packet, is shown in Fig. 3a. Step 1 of the algorithm estimated that 4 feedforward taps per array element and 5 contiguous feedback taps should provide a stable initial configuration for the following stages.

Fig. 3b shows a smoothed graph of the EMSE improvement if a single feedback tap were added at any position; taps will be placed where the curve lies above the threshold shown. Note that the EMSE improvement is not just a scaled version of the impulse response in Fig. 3a, as would be assumed by previous methods which place taps based solely on thresholding the impulse response estimate. To calculate this improvement (without having to perform (2) and (4) for each potential tap), (4) can be rewritten as  $\mathbf{C}^H \mathbf{C} + N_0 \mathbf{I}$  where  $\mathbf{C}$  contains shifted versions of the impulse response. Adding a new feedback tap at lag  $n$  is seen as equivalent to removing a row  $\mathbf{c}^T$  of  $\mathbf{C}$ . Applying the matrix inversion lemma to (5), we arrive at

$$\text{MSE improvement} = \frac{\sigma_d^2 |s[n]|^2}{1 - \mathbf{c}^H \mathbf{R}^{-1} \mathbf{c}} \quad (8)$$

in which  $s[n] = h[n] * a[n]$ , the ISI at time  $n$  caused by the combination of the channel response and feedforward filter.

Taking these new feedback positions into account, step 3 determines the final number of feedforward coefficients, which came out to 10 per array element for this example. This stage requires the greatest number of computations, since (2)-(5) must be calculated for each candidate  $L$ . Straightforward optimizations were made to reduce redundancy in the multiple calculations of  $\mathbf{R}$ , and the search was terminated if (7) increased for two consecutive iterations (which generally produced the global minimum).

With the feedback positions placed properly, the feedforward coefficients can most efficiently concentrate on lowering the ISI at other locations. Fig. 3c shows the ISI from the combined channels and optimal feedforward filters, after step 1 and after step 3. In the latter case (the dashed curve), we see that little attempt is made

to reduce ISI at the feedback tap positions (marked \*\*) because it will be later eliminated by feedback. Instead, the anticausal ISI, for example, can be decreased dramatically.

The feedback tap placement routine was found to be relatively insensitive to the initial condition. However, repeating the feedback procedure in step 4 after adding the feedforward support sometimes adds a few extra taps where needed, without excessive additional cost.

The two critical parameters in the algorithm are the MSE improvement threshold  $\delta$  for setting feedback taps (see Fig. 3b) and the constant  $\tilde{\beta}$  in (7) which determines the stopping criterion for the number of feedforward coefficients. We found that values of 0.0016 and 0.00002 worked well for all of the channel responses and data in the next section. We believe that although these parameters were empirically set, their dependence on EMSE rather than channel response amplitudes will lead to systems that are more robust, in the number of array elements used and in the types of responses they can equalize, than presently used techniques.

#### IV. Experimental Results

The performance of the new fully-automated DFE coefficient placement algorithm (DFE-CPA) is analyzed here using at-sea data from two different data sets, one shallow-water and one deep-water. The first data set contains transmissions that propagated up the shelf break (600-1500 ft water depth) in the Narragansett Bay Operating Areas (NBOA) off the coast of New England in June 1997. Both the transmit and receive platforms were surface research vessels. The second data set was collected in deep water (2500-6000 ft water depth) beyond the NBOA shelf break during the same test. All of the packets contain 6784 symbols (512 training) of mid-frequency (2-5kHz) QPSK data modulated at a rate of 1250 symbols per second. The transmit platform towed the projector array at speeds between 0 and 7 knots while the receive platform drifted.

All signals were received on a 16-element vertical array with approximately 1 foot separation between elements. The array was placed at depths of 60 and 200 meters in the shallow and deep-water segments, respectively. A 13-chip Barker sequence was transmitted a fixed time prior to each QPSK packet to enable packet detection and synchronization. Typical channel responses and their corresponding packet numbers from the shallow and deep sets are shown in Figs. 1 and 2, respectively. These responses are derived from matched-filtering the probe signals on the first element of the receive array. Within these figures are responses of varying complexity and length, some sparse and others continuous.

For comparison, a parameterization technique based on [3] is used in both fully and partially (feedback only with manual optimization of the feedforward taps) automated modes. This baseline algorithm is designed for sparse channels and relies on channel response thresholding for placing taps [4]. In the partially automated mode, a constant number of feedforward taps is maintained over an entire data set. Using the resulting filter support, a fractionally spaced feedforward, LMS-updated adaptive DFE was run on each packet; see [7] for details on the equalizer.

All algorithms were successfully able to equalize multi-element array data (8 channels) from both data sets. A packet is considered successful if its bit error rate (BER) is less than  $10^{-2}$ . Fig. 4 presents the number of shelf break (wedge) packets versus BER for the three tap placement approaches. The new fully-automated DFE-CPA technique shown in Fig. 5a has a 72% success rate on the 304 packets processed; whereas the fully-automated version of the baseline technique (FABT) shown in Fig. 5b is successful on only 56% of the packets. However, the partially-automated (feedback only) baseline technique's (PABT's) performance (69%) shown in Fig. 5c is comparable to that of DFE-CPA. For the deep-water data set, packet success rates of 97% for DFE-CPA, 88% for FABT, and 98% for PABT were achieved. So, in comparing the two fully-automated approaches, DFE-CPA provides a distinct performance advantage across both data sets.

Figs. 5 and 6 display the total number of filter parameters by packet for the wedge and deep data, respectively, as determined by the two fully automated approaches. Figs. 5a and 6a present the successful packets; whereas Figs. 5b and 6b present packets with  $BER > 10^{-2}$ . In general, using DFE-CPA results in fewer overall taps independent of packet success. In some cases, the reduction in taps is by a factor of 2 or more. However, FABT generally identifies fewer taps when the channel response is clearly sparsed (see Figs. 1a and 2h) at the beginning of the shallow-water data set and at the end of the deep-water data set. This result is expected since FABT is designed for sparse channels.

Though not shown in Figs. 5 and 6, PABT used between 800 and 1000 total parameters for each packet with the number of feedforward taps per channel kept constant at 100.

Another goal for a tap placement algorithm would be computational efficiency. With little attempt at code optimization, computations for DFE-CPA are on the same order of magnitude as the demodulation and Doppler resampling stages of the receiver.

## V. Summary

A new fully-automated tap placement algorithm is described and tested in this paper. The technique facilitates equalization of multichannel underwater acoustic data and the results indicate robustness performance in decoding a diverse class of channel responses. This appears to arise from taking some advantage of the interplay between feedforward and feedback taps, as well as incorporating multi-element array data through the use of estimated MSE calculations.

The algorithm often chooses less filter coefficients than the existing automated approach. Both techniques depend heavily on an impulse response estimate, so better channel identification may result in more consistent estimates. More study of the feedforward tap-placement stopping criterion may also be warranted.

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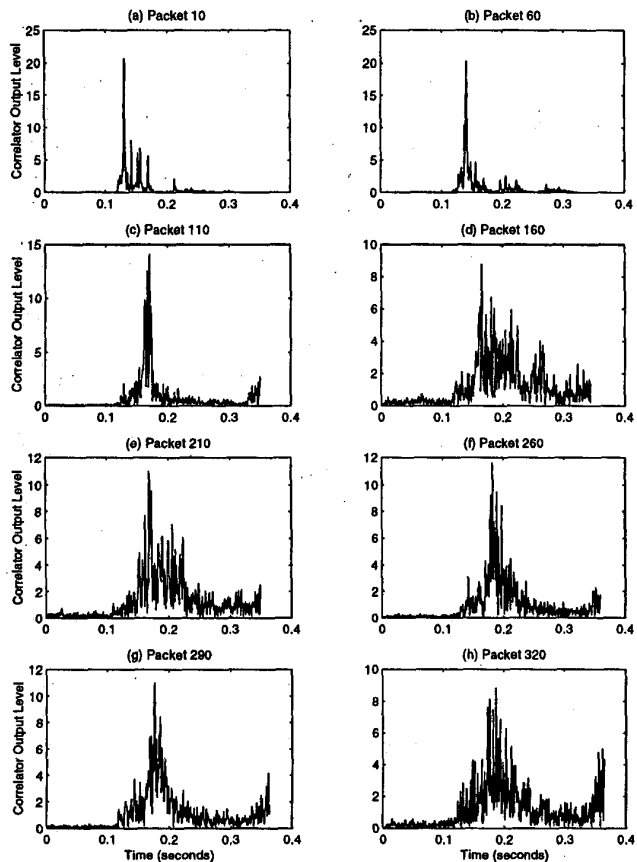


Fig. 1: NBOA shelf break (wedge) channel responses

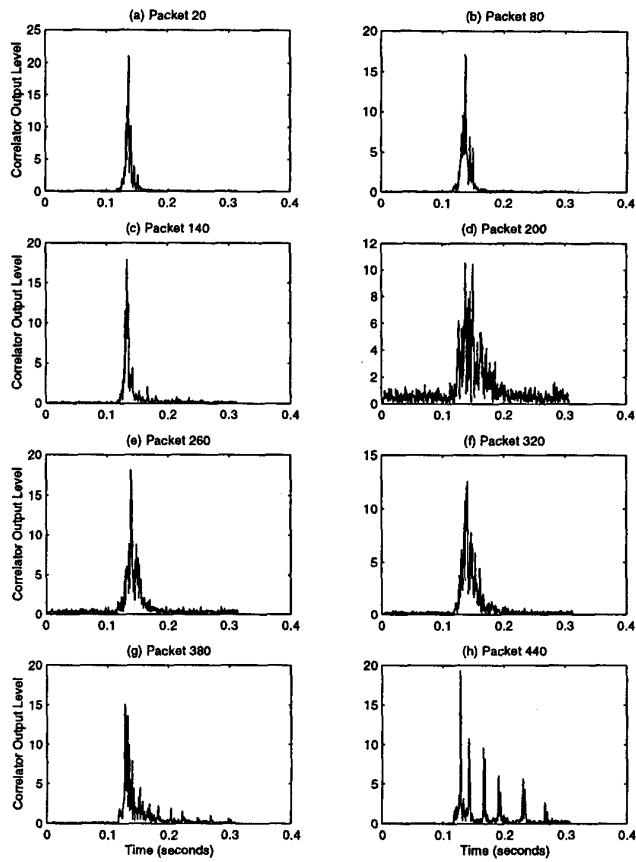


Fig. 2: NBOA deep-water channel responses

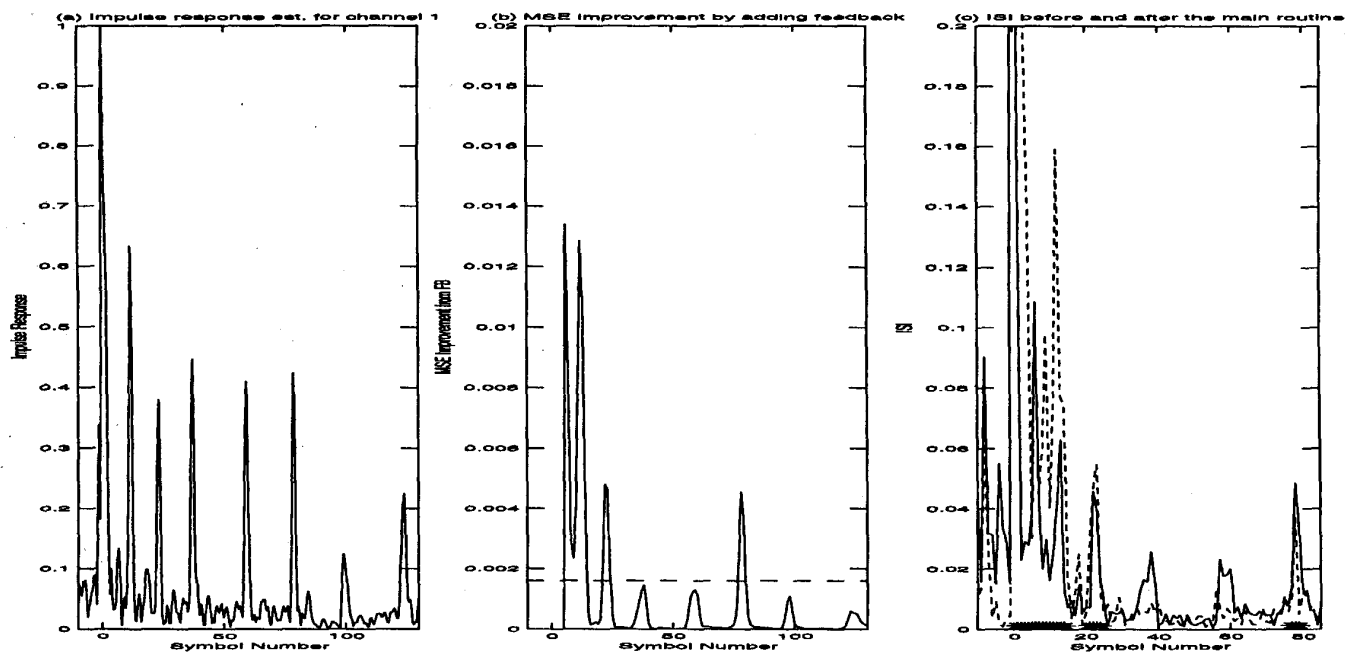


Fig. 3: Algorithm illustration for sparse channel response data

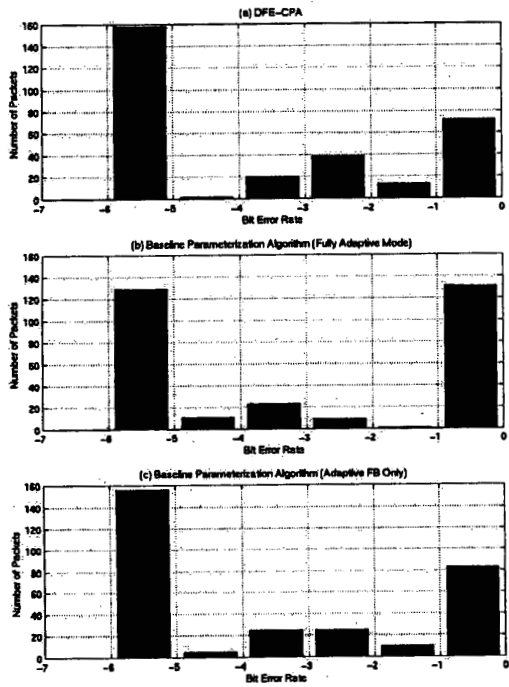


Fig. 4: Shelf break (wedge) data BER performance

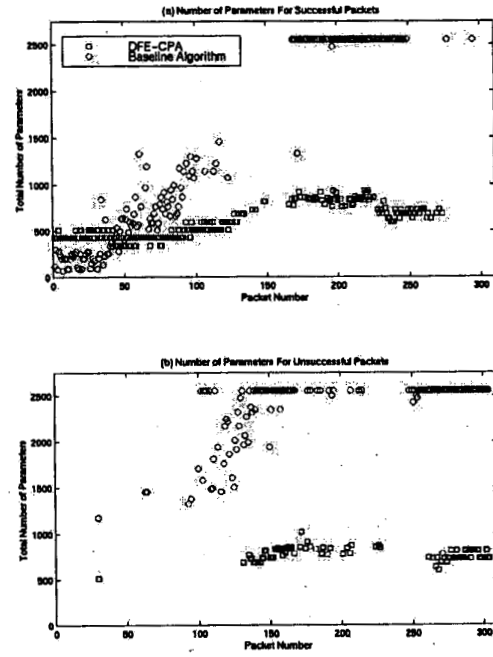


Fig. 5: Number of parameters per packet (wedge)

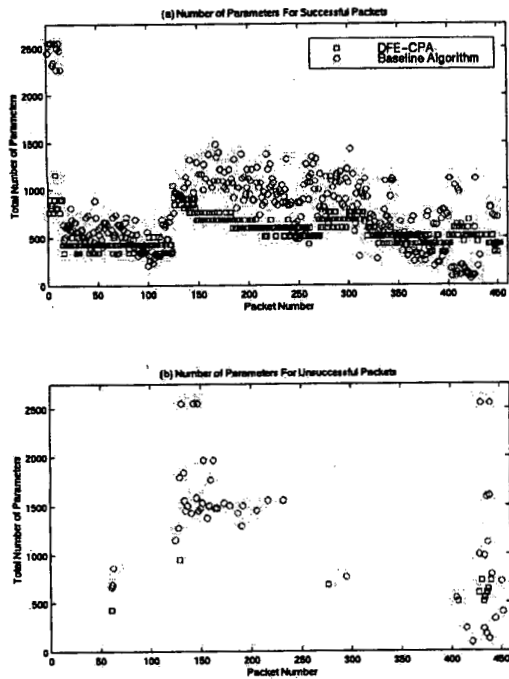


Fig. 6: Number of parameters per packet (deep)