A SEQUENTIAL BAYESIAN BEAMFORMER FOR GAUSS-MARKOV SIGNALS

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ABSTRACT

A Bayesian approach to beamforming is used to derive a sequential adaptive beamformer for estimating Gauss-Markov signals when the source direction-of-arrival (DOA) is uncertain. The DOA is assumed to be randomly selected from a discrete set of candidate directions, with a known probability mass function (PMF). Through a development similar to that of Bell, et al.[1], for i.i.d. sources, the resulting estimator becomes a weighted-combination of Kalman estimators for the source, where the observations for each estimator are retrieved using an MVDR beamformer for each of the candidate DOA's and where the relative weighting is proportional to the likelihood of the DOA given the observed data so far. Aspects of the proposed beamformer, such as robustness to DOA and asymptotic estimation performance are compared with conventional MVDR-based approaches.

1. INTRODUCTION

This paper introduces a sequential adaptive beamformer for estimating Gauss-Markov signals when the source DOA is uncertain. The problem formulation is similar to [1] in which the authors explore the use of a Bayesian approach to enhance DOA robustness for the problem of estimating a temporally uncorrelated signal source. In practice, most source signals are more realistically modeled by correlated source such as the Gauss-Markov signal. Such a signal model accounts for the correlation across time and is often used to model a broad class of FIR channels in the area of underwater acoustics, speech processing and communications. By exploiting the correlation characteristics of Gauss-Markov signals, sequential estimation can be performed instead of the Simple Matrix Inversion (SMI) method. In this paper, we develop a sequential adaptive beamformer that is capable of estimating a first-order Gauss-Markov signal using an iterative algorithm. The proposed beamformer has the form of a Kalman estimator and can operate without accurate information about the DOA of the desired signal.

2. MATHEMATICAL MODEL

A standard narrowband plane wave beamforming model is used for a set of M bandpass signals with known center frequency w_0 arriving at a uniform linear array of N sensors. The array outputs are sampled at times $t = 0, T, 2T, \ldots$ such that the received data is represented by the vector $\vec{x}[n] = \vec{x}(nT)$ as a superposition of the array responses driven by the M incoming signals and noise:

$$\vec{x}[n] = \sum_{m=0}^{M-1} \vec{a}(\theta_m) s_m[n] + \vec{w}[n] \tag{1}$$

where $\vec{a}(\theta_m)$ is the Nx1 array steering vector in the direction θ_m, $s_m[n], m = 0, \ldots, M-1$ are the M equivalent discrete-time baseband source signals at time $t = nT$ and $\vec{w}[n]$ is independent zero-mean Gaussian noise with covariance $\sigma_w^2 \mathbf{I}$.

One of the source signals, $s_0[n]$ is the desired signal and is assumed to be a stationary, zero-mean Gauss-Markov process generated by the following recursion,

$$s_0[n] = b\, s_0[n-1] + u[n], \tag{2}$$

where b is assumed known and where $u[n]$ is a stationary white Gaussian noise sequence of variance σ_u^2. The variance of $s_0[n]$ is then equal to

$$\sigma_s{}^2 = E[|s_0[n]|^2] = \frac{\sigma_u{}^2}{1-|b|^2}. \tag{3}$$

The remaining $M-1$ incoming signals are referred as interference. Each of the interference signals is assumed to be zero-mean Gaussian noise uncorrelated with both the noise $\vec{w}[n]$ and the desired signal. Rewriting (1), the observation vector $\vec{x}[n]$ takes the form

$$\begin{aligned}\vec{x}[n] &= \vec{a}(\theta_0)s_0[n] + \sum_{m=1}^{M-1} \vec{a}(\theta_m)s_m[n] + \vec{w}[n] \\ &= \vec{a}(\theta_0)s_0[n] + \vec{v}[n]\end{aligned} \tag{4}$$

where $\vec{v}[n]$ is the noise-plus-interference component and is uncorrelated with the desired signal $s_0[n]$.

In what follows, we denote $s_0[n]$ and θ_0 by $s[n]$ and θ, respectively, to represent the desired signal and its DOA. We also denote $\vec{x}[n]$ by $\vec{x}_n$ for simplicity.

3. MMSE OPTIMAL GAUSS-MARKOV ESTIMATION

To estimate the desired signal from the received data, we look for an estimate that minimizes the mean squared error (MSE). Let X_k be a collection of k snapshots of the received data $\vec{x}_1, .., \vec{x}_k$. It is well-known that the MMSE optimal estimate of $s[k]$ based upon X_k is the conditional mean of the desired signal $s[k]$, given by

$$\begin{aligned}\hat{s}_{\text{MMSE}}[k] &= \arg\min E[|s[k] - \hat{s}[k]|^2 | X_k] \\ &= E[s[k]|X_k].\end{aligned} \tag{5}$$

0-7803-7551-3/02/$17.00

When $s[k]$ is Gauss-Markov, a Kalman estimator can be used to produce the MMSE optimal estimate of $s[k]$ [2], [3]. The recursive update of the Kalman estimator depends on two pieces of information about the incoming signal: the Gauss-Markov memory parameter b, and the DOA of the desired signal θ. Provided that these parameters are known, the recursive update steps can be written as

$$\hat{s}_{k/k} = b(1 - G_k{}^H \vec{a}(\theta))\hat{s}_{k-1/k-1} + G_k{}^H \vec{x}_k \tag{6}$$

where

$$\hat{s}_{k/k} \triangleq E[s[k]|X_k; \theta, b] \tag{7}$$

$$G_k = (e_k^2\vec{a}(\theta)\vec{a}(\theta)^H + \Sigma_v)^{-1}e_k^2\vec{a}(\theta) \tag{8}$$

$$e_k^2 = |b|^2(1 - G_{k-1}^H\vec{a}(\theta))e_{k-1}^2 + \sigma_u^2, \quad e_0^2 = 0, \tag{9}$$

where $\hat{s}_{k/k}$ stands for the MMSE estimate of $s[k]$ based on observations from $\vec{x}_1$ up to $\vec{x}_k$. By using the Matrix Inversion Lemma [4], we can find the expressions for $G_k^H\vec{a}(\theta)$ and $G_k^H\vec{x}_k$ in terms of e_k^2, $\vec{a}(\theta)$, and Σ_v, in effect eliminating G_k from the recursion. Equations (6) - (9) can now be simplified as

$$\hat{s}_{k/k} = \left(\frac{b}{1+\beta(\theta)e_k^2}\right)\hat{s}_{k-1/k-1} + \left(\frac{\beta(\theta)e_k^2}{1+\beta(\theta)e_k^2}\right)\left(\frac{\Sigma_v^{-1}\vec{a}(\theta)}{\beta(\theta)}\right)^H \vec{x}_k \tag{10}$$

where,

$$\beta(\theta) \triangleq \vec{a}(\theta)^H\Sigma_v^{-1}\vec{a}(\theta). \tag{11}$$

$$e_k^2 = \frac{|b|^2e_{k-1}^2}{1+\beta(\theta)e_{k-1}^2} + \sigma_u^2, \quad e_0^2 = 0 \tag{12}$$

The current estimate $\hat{s}_{k/k}$ depends on two terms. The first term is the recursive term from the past estimate, while the second term is the innovation contributed by the current observation. The term $\beta(\theta)$ is inversely proportional to the power of the noise. When the noise is weak, $\beta(\theta)$ is large, the contribution by the recursion becomes weaker and the current estimate relies more on the observation. When the noise is strong, $\beta(\theta)$ is small, the estimate puts less emphasis on the observation and more on the past estimate as the observation is less reliable.

The term e_k^2 represents the prediction error of $s[k]$, i.e. $e_k^2 = E[(s[k] - \hat{s}_{k/k-1})^2|X_{k-1}]$, where $\hat{s}_{k/k-1}$ is the MMSE prediction of $s[k]$ given X_{k-1}. When e_k^2 is small, the desired signal is predictable, and the current estimate puts more weight on the past estimate and less on the current observation. When the signal is hard to predict, e_k^2 becomes large, and the recursive term becomes small. As a result, more emphasis is put on the observation.

An ideal minimum variance distortionless response (MVDR) beamformer is present in the innovation term. The retrieval of the observations is done by applying an ideal MVDR beamformer in the optimal pointing direction. The MVDR beamformer that points to θ has the form

$$\vec{w}_{MV}(\theta) = \frac{\Sigma_v^{-1}\vec{a}(\theta)}{\vec{a}(\theta)^H\Sigma_v^{-1}\vec{a}(\theta)}. \tag{13}$$

A MVDR beamformer is found to be identical to a minimum power distortionless response beamformer that has the following form

$$\vec{w}_{MP}(\theta) = \vec{w}_{MV}(\theta) = \frac{\Sigma_x^{-1}\vec{a}(\theta)}{\vec{a}(\theta)^H\Sigma_x^{-1}\vec{a}(\theta)}. \tag{14}$$

The Kalman estimator in (10) is derived under the condition that θ is equal to the true DOA of the desired signal. However, if the estimator points to a direction that is different from the true DOA, the estimator will not produce an optimal estimate. In many situations, we need to estimate a signal that comes from an unknown direction. Even if the true DOA is known, a small change in DOA (pointing error) may cause significant degradation in performance. A Bayesian beamformer that is able to perform well under uncertainty in DOA is developed to deal with these problems.

4. DOA ROBUSTNESS USING A BAYESIAN APPROACH

We assume the DOA of the desired signal to be a discrete random variable with a priori probability $q(\theta)$, where θ is defined on a set of discrete candidate DOA's $\{\theta_1, ..., \theta_L\}$. We can now expand the conditional mean of $s[k]$ such that

$$\begin{aligned}\hat{s}_{\text{MMSE}}[k] &= E[s[k]|X_k] = E_\theta[E[s[k]|X_k, \theta]] \\ &= \sum_{i=1}^{L} p(\theta_i|X_k)E[s[k]|X_k, \theta_i].\end{aligned} \tag{15}$$

The inner expectation term represents the MMSE estimate of $s[k]$ under the hypothesis that the observations and θ_i are given. This term is the same as the MMSE estimate in (10) in which the recursive steps are determined with known θ. Substituting (10) into (15), we obtain

$$\begin{aligned}\hat{s}_{\text{MMSE}}[k] &= \sum_{i=1}^{L} p(\theta_i|X_k)\hat{s}_{k/k,\theta_i} \\ &= \sum_{i=1}^{L} p(\theta_i|X_k)\left(\frac{b}{1+\beta(\theta_i)e_k^2}\right)\hat{s}_{k-1/k-1} \\ &\quad + \sum_{i=1}^{L} p(\theta_i|X_k)\left(\frac{\beta(\theta_i)e_k^2}{1+\beta(\theta_i)e_k^2}\right)\left(\frac{\Sigma_v^{-1}\vec{a}(\theta_i)}{\beta(\theta_i)}\right)^H \vec{x}_k.\end{aligned} \tag{16}$$

When no interference signal is present within the range of candidate DOA's, the term $\beta(\theta_i)$ is nearly a constant over all θ_i and can be approximated by $N/\sigma_w{}^2$ [1]. Under this assumption, (16) can be approximated by

$$\begin{aligned}\hat{s}_{\text{MMSE}}[k] \approx &\left(\frac{b}{1+(N/\sigma_w^2)e_k^2}\right)\hat{s}_{k-1/k-1} \\ &+ \left(\frac{(N/\sigma_w^2)e_k^2}{1+(N/\sigma_w^2)e_k^2}\right)\sum_{i=1}^{L} p(\theta_i|X_k)\left(\frac{\Sigma_v^{-1}\vec{a}(\theta_i)}{\beta(\theta_i)}\right)^H \vec{x}_k.\end{aligned} \tag{17}$$

The innovation term consists of terms of the form

$$\sum_{i=1}^{L} p(\theta_i|X_k)\left(\frac{\Sigma_v^{-1}\vec{a}(\theta_i)}{\beta(\theta_i)}\right)^H \vec{x}_k, \tag{18}$$

which is a weighted sum of MVDR beamformers over a range of DOA's, and the contribution by each beamformer is governed by the corresponding a posteriori probability $p(\theta_i|X_k)$. As a result, the Bayesian beamformer is also a Kalman estimator that consists of a recursive term and an innovation term in which observations are retrieved by MVDR beamformers weighted over a range of DOA's.

The a posteriori probability $p(\theta|X_k)$ represents the probability of the DOA of the desired signal, θ, given the observations up to time k. For each θ_i, we can write the a posteriori probability as

$$p(\theta_i|X_k) = \frac{p(\theta_i|X_{k-1})p(\vec{x}_k|\theta_i, X_{k-1})}{p(\vec{x}_k|X_{k-1})}. \tag{19}$$

To obtain $p(\vec{x}_k|\theta_i, X_{k-1})$, we combine (2) and (4) and write $\vec{x}_k$ recursively:

$$\vec{x}_k = b\vec{x}_{k-1} + \vec{n}[k] - b\vec{n}[k-1] + \vec{a}(\theta_i)u[k]. \tag{20}$$

The above expression conveys that $\vec{x}_k$ is a Gaussian random vector with mean vector $b\vec{x}_{k-1}$ and covariance $\bar{\Sigma}_x$ that has the form

$$\Sigma_x = \sigma_u^2\vec{a}(\theta_i)\vec{a}(\theta_i)^H + (1+|b|^2)\Sigma_v \tag{21}$$

where Σ_v is the noise-plus-interference covariance matrix. Based on the condition that $\vec{x}_{k-1}$ is known, we have

$$\begin{aligned}
p(\vec{x}_k|\theta_i, X_{k-1}) &= p(\vec{x}_k|\theta_i, \vec{x}_{k-1}) \\
&= \frac{1}{\pi^N|\bar{\Sigma}_x|}\exp(-\vec{\Delta}_k^H\bar{\Sigma}_x^{-1}\vec{\Delta}_k) \\
&= \left\{\frac{1}{\pi^N(1+|b|^2)|\Sigma_v|(1+\sigma_s{}^2\beta(\theta_i))}\right\} \\
&\cdot \exp\left\{-\frac{\vec{\Delta}_k^H\Sigma_v^{-1}\vec{\Delta}_k}{1+|b|^2}\right\} \\
&\cdot \exp\left\{\frac{1}{1+|b|^2}\left(\frac{\sigma_u{}^2\beta(\theta_i)^2}{1+|b|^2+\sigma_u{}^2\beta(\theta_i)}\right)\right. \\
&\left.\cdot\left(\frac{\vec{a}(\theta_i)^H\Sigma_v{}^{-1}}{\beta(\theta_i)}\vec{\Delta}_k\vec{\Delta}_k^H\frac{\Sigma_v{}^{-1}\vec{a}(\theta_i)^H}{\beta(\theta_i)}\right)\right\}
\end{aligned} \tag{22}$$

where,

$$\vec{\Delta}_k \triangleq \vec{x}_k - b\vec{x}_{k-1}. \tag{23}$$

As mentioned previously, the term $\beta(\theta_i)$ can be approximated by the constant N/σ_w^2. Substituting (22) into (19) and collecting the terms that are independent of θ_i, the a posteriori probability can be expressed recursively by the following approximation:

$$p(\theta_i|X_k) \approx c_k \cdot p(\theta_i|X_{k-1})\exp(\gamma|\vec{w}_{MV}(\theta_i)^H\vec{\Delta}_k|^2) \tag{24}$$

$$p(\theta_i|X_0) = q(\theta_i), \tag{25}$$

where

$$\gamma = \frac{1}{1+|b|^2}\left(\frac{\sigma_u{}^2(N/\sigma_w^2)^2}{1+|b|^2+\sigma_u{}^2(N/\sigma_w^2)}\right) \tag{26}$$

and c_k is a normalization factor [1]. Both γ and c_k are independent of the candidate DOA θ_i. We initialize the a posteriori probabilities by the a priori probabilities of the DOA, $q(\theta_i)$, which carry prior knowledge about the distribution of the DOA's before any observation is received.

Expression (24) can be viewed as a recursive product of k exponentials, where the strength of each exponential is determined by the term $|\vec{w}_{MV}(\theta_i)^H\vec{\Delta}_k|^2$ and parameter γ. The first term $|\vec{w}_{MV}(\theta_i)^H\vec{\Delta}_k|^2$ is directional oriented, and it puts more weight on θ_i when θ_i matches closely with the true DOA θ. Thus, as k goes to infinity, the Bayesian beamformer converges to one DOA by placing all of the weight in the mixture in the mostly correct direction. The parameter γ, which depends on the signal and noise power, amplifies the spatial variation of the a posteriori probability and in effect controlling the rate of the convergence. Figure 1 shows the convergence of 6 candidate points. As k approaches infinity, one candidate point converges to 1, while the rest of them vanish to zero.

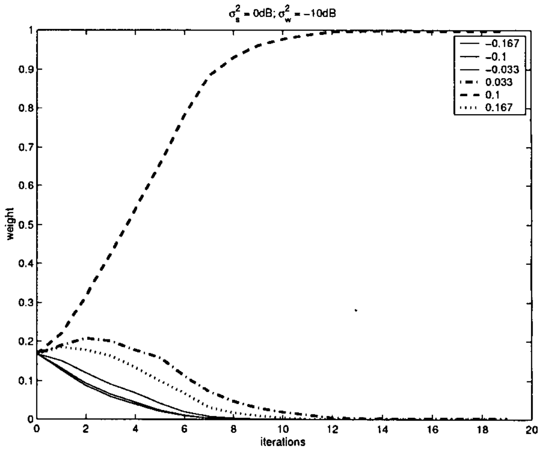

Fig. 1. The convergence of 6 candidate points averaged across 50 trials. True DOA at 0.09.

The recursive update step for the Bayesian beamforming algorithm is derived from (17). The resulting Bayesian beamformer has the form of a Kalman estimator and is given by

$$\begin{aligned}
\hat{s}_{\text{MMSE}}[k] &\approx \left(\frac{b}{1+(N/\sigma_w^2)e_k^2}\right)\hat{s}_{k-1/k-1} \\
&+ \left(\frac{(N/\sigma_w^2)e_k^2}{1+(N/\sigma_w^2)e_k^2}\right)\sum_{i=1}^{L}p(\theta_i|X_k)\vec{w}_{MV}(\theta_i)^H\vec{x}_k,
\end{aligned} \tag{27}$$

where the update steps for e_k^2 and $p(\theta_i|X_k)$ are described by (12) and (24) respectively. The form of the MVDR beamformer $w_{MV}(\theta_i)$ is given in (13).

5. IMPLEMENTATION

To implement the Bayesian beamformer in practice, either the parameters Σ_x, σ_u^2 and σ_w^2 must be assumed known, or they need to be estimated. The covariance matrix Σ_x carries the statistical information of the incoming data. At each iteration k, we estimate this term by the k-sample covariance matrix $\hat{\Sigma}_k$ that has the form

$$\hat{\Sigma}_k = \frac{1}{k}\sum_{i=1}^{k}\vec{x}_i\vec{x}_i^H. \tag{28}$$

As k approaches infinity, the sample covariance $\hat{\Sigma}_k$ converges to the true data covariance Σ_x under stationary and ergodic assumptions. The optimal MVDR beamformer in (15) can be implemented by replacing Σ_x by its estimate $\hat{\Sigma}_k$. However, this substitution can induce a distorted main beam and high sidelobes. To alleviate this effect, diagonal loading is incorporated to the sample covariance matrix to produce a beampattern that has smaller sidelobes [5]. This is done by adding an identity matrix to the covariance matrix at each update such that

$$\hat{\Sigma}_{k,\mathrm{DL}} = \hat{\Sigma}_k + \sigma_k^2 \mathbf{I}. \quad (29)$$

where σ_k^2 is the diagonal loading level at the kth iteration. A high loading level improves the beam shape and sidelobe behavior at the expense of reduced nulling capability against weak interference [5]. To obtain the best result, the loading levels are chosen to be smaller than the interference power and larger than the noise power.

Since all computations involved in this problem require only the inverse of the data covariance matrix Σ_x^{-1}, an efficient recursive algorithm is developed to compute the inverse of the sample covariance matrix with diagonal loading. First, we define the loading level at the kth iteration recursively by

$$\sigma_k^2 = \frac{k-1}{k}\sigma_{k-1}^2, \quad (30)$$

$$\sigma_1^2 = \sigma_{\mathrm{DL}}^2 \quad (31)$$

where σ_{DL}^2 is the preset loading level. Using this substitution and the Matrix Inversion Lemma, the recursive update step of the inverse of the sample covariance matrix $\hat{\Sigma}_{k,\mathrm{DL}}^{-1}$ can be written as

$$\begin{aligned} \hat{\Sigma}_{k,\mathrm{DL}}^{-1} &= \left(\frac{1}{k}\sum_{i=1}^{k} \vec{x}_k \vec{x}_k^H + \sigma_k^2\right)^{-1} \\ &= \frac{k}{k-1}\left(\hat{\Sigma}_{k-1,\mathrm{DL}}^{-1} - \frac{\hat{\Sigma}_{k-1,\mathrm{DL}}^{-1}\vec{x}_k\vec{x}_k^H\hat{\Sigma}_{k-1,\mathrm{DL}}^{-1}}{k-1+\vec{x}_k^H\hat{\Sigma}_{k-1,\mathrm{DL}}^{-1}\vec{x}_k}\right), \quad (32) \\ \hat{\Sigma}_{1,\mathrm{DL}}^{-1} &= (\vec{x}_1\vec{x}_1^H + \sigma_1^2\mathbf{I})^{-1} \quad (33) \end{aligned}$$

which provides a computationally efficient way to compute the sample covariance matrix based on sequential update.

To estimate the power term σ_u^2 from an uncertain direction, we incorporate the Bayesian approach with the minimum variance spatial spectral estimate in [6] such that

$$\begin{aligned} \hat{\sigma}_u^2 &= (1-|b|^2)E[|s[n]|^2|X_k] = E_\theta[E[|s[k]|^2|X_k,\theta]] \\ &= (1-|b|^2)\sum_{i=1}^{L} p(\theta_i|X_k)E[|s[k]|^2|X_k,\theta_i] \\ &= (1-|b|^2)\sum_{i=1}^{L} p(\theta_i|X_k)(\vec{a}(\theta_i)^H\hat{\Sigma}_{k,\mathrm{DL}}^{-1}\vec{a}(\theta_i))^{-1}. \quad (34) \end{aligned}$$

The noise term σ_w^2 is estimated by the reciprocal of the largest eigenvalue of $\hat{\Sigma}_{k,\mathrm{DL}}^{-1}$, which is equivalent to the smallest eigenvalue of $\hat{\Sigma}_{k,\mathrm{DL}}$.

6. SIMULATION

We simulate the Bayesian beamformer and evaluate its performance under different scenarios. In the first subsection, we compare the Bayesian beamformer to the conventional beamformer with fixed pointing direction and demonstrate its enhanced robustness to DOA. In the second subsection, we examine the sequential behavior of the beamformer and evaluate its asymptotic performance.

The signal characteristics are described as follow: The desired signal is a Gauss-Markov process with memory parameter (b) of 0.5. A strong interferer, whose power is 20dB, arrives at the sensor array at $\theta = -0.4$. The beamformer receives observations up to time k=300. The number of sensors (N) is 7.

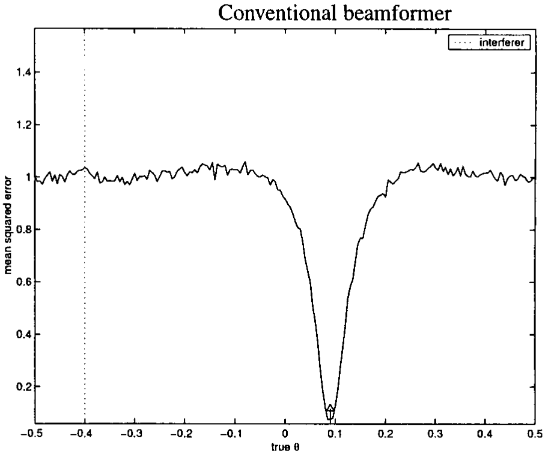

Fig. 2. Mean squared error versus true DOA averaged across 50 trials.

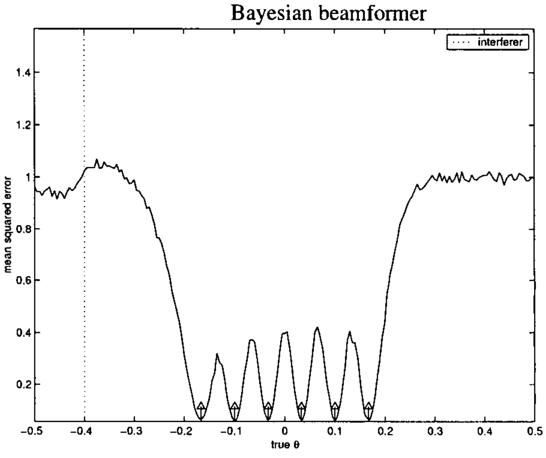

Fig. 3. Mean squared error versus true DOA averaged across 50 trials.

The Bayesian beamformer is set up with the following parameters: Half-wavelength spacing and a Uniform Linear Array (ULA) are used. The number of candidate DOA's (L) is chosen to be 6, and the candidates DOA's are located at {-0.167, -0.1, -0.033, 0.033, 0.1, 0.167} respectively. The a priori distribution $q(\theta)$ is assumed to be uniformly distributed over the 6 candidate DOA's. The preset diagonal loading level is 10dB. The parameters σ_u^2, σ_w^2 and Σ_x are estimated from the data using the methods described previously, and the effect on performance appears to be marginal.

6.1. Test of Robustness

To test the robustness of the proposed beamformer, we consider the case when the desired signal impinges upon the sensor array

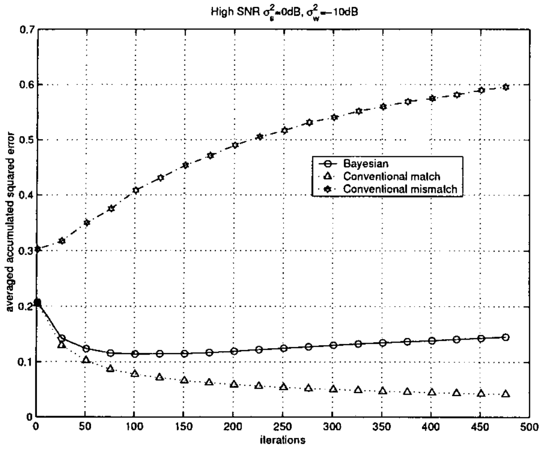

Fig. 4. Averaged accumulated squared error at SNR of 10dB averaged across 50 trials.

at various directions ranging from -0.5 to 0.5. The signal power is 0dB and the noise power is -10dB. For each direction, the desired signal is estimated and the corresponding mean squared error is computed. The same simulation is performed 50 times and the averaged result is plotted.

First, we consider the conventional beamformer introduced in (10)-(12) and have it pointed to the direction $\theta = 0.09$. The mean squared error versus true DOA is shown in Figure 2 where the arrow indicates the pointing direction. The mean squared error is at minimum when the pointing direction matches with the true DOA value. The mean squared error increases when there is a mismatch between the true DOA and the pointing direction.

The same simulation is performed with a Bayesian beamformer. The plot is shown in Figure 3 and the candidate DOA's are indicated by arrows. The Bayesian beamformer suppresses the mean squared error within the range of candidate DOA's within -0.167 and 0.167. This allows the beamformer to operate without accurate information about the source DOA. The increase in error for directions between candidate DOA's arises due to the discrete nature of the candidates chosen. This can be suppressed by either using a more finely chosen set of DOAs or, perhaps, by considering all possible convex combinations.

6.2. Asymptotic Estimation Performance

In this subsection, we examine the sequential behavior and the asymptotic estimation performance of three sequential beamformers: the Bayesian beamformer, the conventional beamformer with mismatched pointing direction, and the conventional beamformer with perfectly matched pointing direction. We plot the mean squared estimation error versus iteration for each beamformer. The true DOA is 0.09. The mismatched beamformer has pointing direction of 0.123. The operational SNR is 10dB.

In Figure 4, we see that, from the beginning, the Bayesian beamformer always outperforms the conventional beamformer with mismatched pointing direction. From the start, this is due to the initial wide beam of the Bayesian beamformer, guaranteeing the retrieval of the desired signal. Eventually, the Bayesian beamformer converges to the single beamformer with pointing direction of 0.1, thus produces a slightly larger error than the conventional beamformer with perfect pointing direction (of 0.09).

7. CONCLUSION

This paper explores the use of a Bayesian approach for the problem of estimating Gauss-Markov signals when the DOA is uncertain. The proposed beamformer is a weighted sum of Kalman estimators derived at a set of hypothesized candidate DOA's. The weighting coefficients are assigned by the a posteriori distribution conditioned on the received observations. A computationally efficient recursive algorithm is introduced and analyzed. Simulation results illustrate that the Bayesian beamformer enhances robustness to DOA uncertainty within a predetermined range of DOA's.

8. REFERENCES

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