A Class of Frequency-Hop Patterns
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Abstract — We present a new class of frequency-hop patterns over GF(q) with period \( m \cdot n \) where \( m \) is a divisor of \( q - 1 \) and \( n \) is a divisor of \( q + 1 \). The Hamming correlation properties of these frequency-hop patterns are nearly optimal, and the patterns also have other properties that make them attractive for use in frequency-hop communication systems.

I. INTRODUCTION
The bandwidth allotted to a frequency-hop (FH) communication system is divided into nonoverlapping frequency bands called slots. The carrier frequency of a transmitter is changed (hopped) periodically from slot to slot in accordance with a hopping pattern. During the time that the carrier is in a slot, conventional narrowband signaling techniques are used to transmit information. Since each transmitter uses only one slot at any time, other transmitters (with different hopping patterns) can be using other slots. However, because of lack of synchronism, two transmitter signals may try to use the same slot at same time and such collisions cause severe signal degradation and many transmission errors.

A hopping pattern in a FH communication system with \( q = p^r \) slots (\( p \) a prime) may be thought of as a vector \( \mathbf{x} = (x_0, x_1, \ldots, x_{N-1}) \) of \( N \) elements of GF(\( q \)). Let \( x \) and \( y \) denote two hopping patterns. In order to minimize the number of collisions between \( x \) and \( y \), the Hamming distance between the various cyclic shifts of these patterns should be as large as possible. Thus, it should not be too surprising that cyclic MDS codes have been used for constructing hopping patterns. Two known constructions are due to Reed and (separately) to Solomon who used low-rate cyclic \( (q - 1, k, q - k) \) Reed-Solomon codes. When \( k = 2 \), both constructions produce the same set of \( q \) hopping patterns of period \( N = q - 1 \) with at most \( k - 1 \) collisions. When \( k > 2 \), the two constructions produce different sets of patterns but with comparable parameters. A different construction, due to Lempel and Greenberger uses an \( l \)-stage maximal-length linear feedback shift register over GF(\( p \)) to produce \( p^r \) hopping patterns, \( r \leq l \), of period \( N = p^l - 1 \) over GF(\( q \)) with at most \( p^{r-l} \) collisions. Details can be found in [3].

II. THE NEW CONSTRUCTION
The class of hopping patterns introduced in this paper is of length between \( q + 1 \) and \( q^2 - 1 \), and is thus comparable to sets of Lempel-Greenberger hopping patterns of period \( p^l - 1 \), \( r < l \leq 2r \). The new class can be thought of as being obtained from the pseudocyclic MDS codes studied in [1].

Thus, it inherits several good properties of MDS codes. We use the codewords of an \((n, k, d)\) pseudocyclic code \( C \) modulo \((z^n - a)\) to construct an \((mn, k, md)\) cyclic code \( C' \) whose codewords \( C' \) are concatenations of the codewords of \( C \). Here, \( n \) is a divisor of \( q + 1 \) and \( m \) is the multiplicative order of \( a \) in \( GF(q) \). The cycle representatives of period \( mn \) of the codewords in the \((mn, k, md)\) cyclic code \( C' \) constitute a set of hopping patterns. Obviously, these hopping patterns have period \( N = mn \), and there cannot be more than \( (q^k - 1)/mn \) such hopping patterns. Equally obviously, the maximum number of collisions is \( m(n - d) = m(k - 1) \). Furthermore, these collisions are spread out over the \( mn \) hops with no more than \( k - 1 \) collisions during any \( n \) consecutive hops. We also show that the positions of the collisions during \( n \) successive hops are duplicated during the next \( n \) hops, that is, the collisions occur in the same relative positions each time.

These properties of our new hopping patterns might be exploited in different ways. When collisions occur, they are usually detectable, and the corresponding symbols are erased. If a Reed-Solomon code of length \( n \) is being used for erasure and error correction, then the erasure locator polynomial need not be calculated afresh when decoding each successive received word. Some savings in computation are possible even if the Reed-Solomon code length is not \( n \), since succeeding received words will have the erasures displaced by a fixed amount. Alternatively, adaptive coding and transmission strategies might be feasible. For example, after \( n \) hops, the receivers know which of the succeeding hops will have collisions. If this information is made known via feedback to the transmitters, then adaptive transmission methods, such as all but one transmitter maintaining radio silence during affected hops, might be used to avoid collisions altogether. While this reduces the data rate, such reduction could be ameliorated by increasing the code rate — in the absence of collisions, less powerful error correcting codes can be used. Further details can be found in [2].

REFERENCES