Comments on “An Alternative Derivation for the Signal-to-Noise Ratio of a SSMA System”

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Abstract—The “alternative derivation” proposed in the LETTER referenced in the title is incorrect. The correct derivation gives a result that agrees with previous computations. Hence, the claims and conclusions based on the apparent difference in the results are not valid.

I. INTRODUCTION

The “alternative derivation” and the main result in the above-referenced LETTER by Wei [1] are incorrect. The error in the derivation occurs in the three steps leading to (7) in which the random variable $\tau_k$ is renamed as two different (independent) random variables $\tau_k$ and $\tau_k'$. Put another way, the derivation of (7) equates $E[\rho(\tau_k)h(\tau_k)]$ to $E[\rho(\tau_k')h(\tau_k')]$, which is not valid in this particular instance. The integrand in the right side of (7) should actually read as follows:

$$E[a_k(\alpha - \tau_k)a_k(\beta - \tau_k)b_k(\alpha - \tau_k)b_k(\beta - \tau_k) \times \cos^2(\omega_c \tau_k - \theta_k)] \cdot E[a_k(\alpha)\bar{a}_k(\beta)]d\omega d\theta.$$

Computing expectation with respect to the random variable $\theta_k$ first allows the elimination of the $\cos^2$ factor via (8). Next, note that conditioned on the data sequence $\{b_{(k)}^{(k)}\}$ and chip sequence $\{a_{(k)}^{(k)}\}$, the expected value of $a_k(\alpha - \tau_k)a_k(\beta - \tau_k)b_k(\alpha - \tau_k)b_k(\beta - \tau_k)$ with respect to $\tau_k$ is the continuous-time autocorrelation function of the signal $a_k(t).$ The expectation of this autocorrelation function with respect to the data sequence $\{b_{(k)}^{(k)}\}$ and chip sequence $\{a_{(k)}^{(k)}\}$ is given by (10). Hence, it follows from (9) that

$$\text{var}(y_k(T)) = \frac{P}{4} \sum_{n=0}^{N-1} \int_{-\infty}^{\infty} \left( 1 - \frac{\alpha - \beta}{T_c} \right) d\alpha d\beta \cdot \frac{PN}{2} \int_{0}^{T_c} \left( 1 - \frac{\gamma}{T_c} \right) (T_c - \gamma) d\gamma \cdot \frac{PT^2}{2N}$$

and thus the right side of (14) should be exactly the right side of (17) of [2].

Since there is no discrepancy between the results of [2] and the corrected version of Wei’s result, several claims in Wei’s LETTER are not tenable. For example, it is not true that the result of [2] is approximate, and just a special case of her result for large $N$. Similarly, after correction, her formula no longer “suggests that more users are allowed in a SSMA system in general.”

Wei claims that she computes the ensemble average whereas a time average was computed in [2] and an appeal to stationarity and ergodicity was required to deduce the ensemble average. This is incorrect. To quote from page 798 of [2]:

“We should point out that this signal-to-noise ratio is computed by means of probabilistic averages (expectations) with respect to the phase shifts, time delays, and data symbols.”

Both methods compute the ensemble average—it is just that the expectations are computed in different order in the two derivations. Wei assumes right from the start that the signature sequences are random. On the other hand, in [2], the signal-to-noise ratio (SNR) is first obtained in (15) of [2] for any specified set of signature sequences in terms of certain correlation parameters of the given signature sequences. Equation (17) of [2] is then obtained by replacing the correlation parameters in (15) by their expected values for random sequences.

It is pointed out quite clearly in [2] that (17) is just a minor result that allows the system designer to gauge roughly (without the drudgery of first computing the correlation functions) whether the desired SNR can be achieved with the chosen system parameters. Note that it is not claimed in [2] that the right side of (17) is the SNR for any specific SSMA system, nor is it proposed that the right side of (17) be used as anything other than a rough estimate of the SNR. In fact, it is suggested in [2] that once the system parameters have been chosen appropriately, the exact SNR should be computed using Eq. (15) of [2]. In contrast, Wei’s LETTER not only contains an incorrect computation for the rough estimate of Eq. (17) of [2] but it also appears to suggest that this is indeed the exact value of the SNR of a SSMA system.

II. SUMMARY

In summary, the derivation and the main result in Wei’s LETTER [1] are incorrect, and the claims made therein are baseless.

REFERENCES
