Error Criteria Analysis and Robust Data Fusion
Xing Li, Weiru Fang and Qi Tian
Electronic Engineering Department
Tsinghua University
Beijing 100084, China

ABSTRACT

Except the well-known least mean square error criterion which results in the arithmetic mean operation and the mean absolute value criterion which results in the median operation, other nonlinear operations have not been extensively studied in terms of error criteria in the existing literature. In this paper, the error criteria of the harmonic mean, geometric mean, as well as some other nonlinear operations have been derived. Both simulation and experimental data presented in this paper indicate that these nonlinear operations are more robust than that of the linear arithmetic mean. Therefore, the error criteria analysis method can be used as a general guide line for designing the most appropriate operations for the robust fusion problems.

1. INTRODUCTION

The optimal detection of targets based on multiple observations in a data fusion system is of great importance in many applications. However, an operation can only be regarded as 'optimal' with respect to the specified performance criteria, of which the minimum mean square error criterion is most often used for mathematical convenience. But it has been suggested that this criterion is not appropriate for many problems [1-2]. Unfortunately, other nonlinear operations have not been extensively studied in terms of error criteria, with the exception of the above-mentioned least mean square error criterion which results in the arithmetic mean operation and the least mean absolute value error criterion which results in the median operation [3-4]. In this paper the error criteria of the harmonic mean, geometric mean, as well as some other nonlinear operations have been derived. The error criteria studied in this paper indicate that the harmonic mean, geometric mean and median operations are more robust than the arithmetic mean operation. Both simulation and experimental data are presented in this paper to support the results of the analysis.

2. ANALYSIS AND SIMULATION RESULTS

Suppose a set of observations

\[ x_i = \mu + \eta_i \]  

are given, where \( \mu \) is a location parameter and \( \eta_i, i = 1, \ldots, N \) are zero mean noise components [5-6]. Then for some function

\[ \rho(x, \mu) = 0 \]  

which satisfies the condition \( \rho(\mu, \mu) = 0 \), an estimator \( \hat{\mu} \) of \( \mu \) can be defined which minimizes

\[ \epsilon = \sum_{i=1}^{N} \rho(x_i, \hat{\mu}) \]  

or which equivalently satisfies

\[ \sum_{i=1}^{N} \frac{d}{d\hat{\mu}} \rho(x_i, \hat{\mu}) = 0. \]  

In some special cases, the close form solution \( \hat{\mu} = \tilde{f}(x_1, \ldots, x_i, \ldots, x_N) \) can be obtained. The arithmetic mean, median, harmonic mean and geometric mean are within this category as shown in Table 1 (Note: the observations \( x_i \)'s have to be greater than 0 for the harmonic mean and geometric mean operations). Notice that the error criteria for the arithmetic mean and the median can be found in many references [3-4]. However, almost no literature has discussed the harmonic mean and the geometric mean from the error criterion point of view.

If one infers the right forms of the error criteria, the proof of Table 1 is quite straightforward. Figure 1 shows the error criteria for the arithmetic mean (1), median (2), harmonic mean (3) and geometric mean (4), respectively, where the location parameter (\( \mu \)) is assumed to have the same value for the purpose of comparison. Note that, for those nonlinear operations concerned, the observations which are far from the correct location (\( \mu \)) will make less contribution to producing \( \mu \) as distinct from the arithmetic mean. In these cases the estimated values will be less sensitive to the 'bad' observations (i.e. the observation with large variance), and they are therefore more robust.
Table 1

<table>
<thead>
<tr>
<th>Operation</th>
<th>Error Criterion</th>
</tr>
</thead>
<tbody>
<tr>
<td>Arithmetic mean</td>
<td>$\mu_a = \frac{1}{N} \sum_{i=1}^{N} x_i$</td>
</tr>
<tr>
<td>Median</td>
<td>$\mu_m = \text{med}(x_1, \ldots, x_N)$</td>
</tr>
<tr>
<td>Harmonic mean</td>
<td>$\mu_h = \frac{N}{\sum_{i=1}^{N} \frac{1}{x_i}}$</td>
</tr>
<tr>
<td>Geometric mean</td>
<td>$\mu_g = \left( \prod_{i=1}^{N} x_i \right)^{\frac{1}{N}}$</td>
</tr>
</tbody>
</table>

The operations analyzed above are evaluated using Monte-Carlo method for the two hypotheses detection problem as shown in Figure 2. The simulation parameters are: the number of observations is equal to 6, the observations are assumed to be Gaussian and statistically independent with the mean values $\mu_0 = 0$ (hypothesis 0) and $\mu_1 = 5$ (hypothesis 1), the standard deviations of different observations ($\sigma_i$) are equal to 5 except for the observation 1, where $\sigma_1$ is a variable. The decision threshold is determined using N-P criterion with $P_f = 0.1$. Notice that the absolute value has been taken for the harmonic mean and geometric mean operations in order to satisfy the condition $x_i > 0$. It is clear that the harmonic mean, geometric mean and median operations are more robust than the arithmetic mean operation.

3. A GENERALIZATION OF ERROR CRITERIA

In order to study further the robust property of different error criteria, the generalization of the harmonic mean and the geometric mean have been made in this paper as shown in Table 2 (Note: the observations $x_i$'s have to be greater than 0). The proof of Table 2 is also straightforward by using Equation (4).

Notice that the parameters $p$, $q$, and $r$ can take real values. In terms of the generalized harmonic mean operation, it is interesting to point out that the 1st type is generalized based on the 'error criterion' representation, while the 2nd type is generalized based on the 'operation' representation. However, if $p=1$ and $q=1$, both types will become ordinary geometric mean and if $p=2$ and $q=1$, both types will become arithmetic mean. As regards the generalized geometric mean operation, if $r=0$, it will become ordinary geometric mean. Therefore, the generalization made here actually covers a wide range of operations. Figures 3, 4, 5 show the error criteria for the 1st type and 2nd type generalized harmonic mean operations, as well as the generalized geometric mean operation, respectively.

The computer simulations have also been performed under the same conditions as those described in Section 2. Figure 6 shows the probability of error when the generalized geometric mean operation is applied. It is clear that the performance of $r=0.1$ is better than $r=0.0$ (geometric mean) for $\sigma_1 < 25$, but if $\sigma_1$ increases, smaller or even negative $r$ within a certain range, will be more robust. By comparing with the error criteria function shown in Figure 5, it is of interest to note that for smaller $r$ the observations which are far from the correct location $(a)$

![Fig. 1 The error criteria of the mean, median, harmonic mean and geometric mean operations](image1)

![Fig. 2 The robustness of the mean, median, harmonic mean and geometric mean operations (simulation results)](image2)
### Table 2

<table>
<thead>
<tr>
<th>Generalized Harmonic Mean</th>
<th>Operation</th>
<th>Error Criterion</th>
</tr>
</thead>
<tbody>
<tr>
<td>(1st Type)</td>
<td>[ \mu_{gh1} = \frac{\sum_{i=1}^{N} x_i^{y-1}}{\sum_{i=1}^{N} x_i^{y-2}} ]</td>
<td>[ e_{gh1} = \sum_{i=1}^{N} (x_i y^{y-1} - \mu_{gh1})^2 ]</td>
</tr>
<tr>
<td>(2nd Type)</td>
<td>[ \mu_{gh2} = \left[ \frac{1}{N} \sum_{i=1}^{N} x_i \right]^{\frac{1}{y}} ]</td>
<td>[ e_{gh2} = \sum_{i=1}^{N} ((x_i) y^{y-1} - \mu_{gh2})^2 ]</td>
</tr>
<tr>
<td>Geometric Mean</td>
<td>[ \mu_{ge} = \left[ \prod_{i=1}^{N} x_i^{\frac{1}{y}} \right]^{\frac{1}{N}} ]</td>
<td>[ e_{ge} = \sum_{i=1}^{N} ((x_i) \log(x_i) - \mu_{ge})^2 ]</td>
</tr>
</tbody>
</table>

will make less contribution to producing \( \mu \). Hence, smaller \( r \) is desirable in order to achieve robust property. However, the smaller \( r \) may produce higher probability of error if the characteristics of the observations are similar. Therefore, a trade-off has to be made for a specific application. (It should be pointed out that the above conclusion may not hold unless \( r \) is close to zero. For cases in which the absolute value of \( r \) is not close to zero, further investigations need to be made.) The similar results have also been obtained by computer simulation using the generalized harmonic means (both 1st and 2nd type).

### 4. Ultrasonic Experimental Results

The robust property of the nonlinear operations have also been evaluated by some ultrasonic nondestructive testing experiments. Figure 7 shows a typical ultrasonic signal, where the grain noise totally masks the flaw echo [7]. In order to enhance the SNR, the split spectrum processing technique (SSP) has been applied as shown in Figure 8. This technique creates the frequency diverse multiple observations using a filterbank and forms an output using linear or nonlinear operations. It has been reported that the SNR can be improved using the SSP technique, but some operations are very sensitive to the processing frequency range. However, in the past, no theory has been found which can be used to choose the 'optimal' operation.

Importantly, the error criteria method presented in this paper provides a guide line to design the robust operations for the SSP technique. Figure 9 shows the output SNRs of the split spectrum processing technique versus the processing frequency range. Clearly, the geometric mean operation is the most robust one. The output of the geometric mean operation is shown in Figure 10 where the flaw is clearly visible. The SSP technique using the generalized harmonic mean and the geometric mean operation with different parameters is currently under

![Fig. 3](image1)

**Fig. 3** The error criteria of the generalized harmonic mean operations (1st-type)

![Fig. 4](image2)

**Fig. 4** The error criteria of the generalized harmonic mean operations (2nd-type)

![Fig. 5](image3)

**Fig. 5** The error criteria of the generalized geometric mean operations
5. CONCLUSIONS

In conclusion, the error criteria for some nonlinear operations have been derived in this paper. The simulation and ultrasonic experiments indicate that these nonlinear operations are more robust than the linear arithmetic mean operation. The authors believe that the progress in this area will provide a general guide line for designing the most appropriate operations for the robust fusion problems.

ACKNOWLEDGMENTS

This project is sponsored by the Chinese State Educational Commission.

REFERENCES