Tradeoff Optimality of D-BLAST

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October 27, 2004

Fading MIMO Channel

◇ $\mathbf{y} = \mathbf{H}\mathbf{x} + \mathbf{w}$

◇ Entries of $\mathbf{H}$ have a joint distribution.

– slow fading, coherent

Fading MIMO Channel

◇ $\mathbf{y} = \mathbf{H}\mathbf{x} + \mathbf{w}$

◇ Focus of this talk

Short block-length communication at high SNR

Rate and Probability of Error

- ◇ Tradeoff between rate R , and probability of error $\mathbb{P}_e$
- ◇ **Outage:** Given a rate and SNR:

$$\min_{\mathbb{P}_{\mathbf{x}}} \mathbb{P}(\{ \mathbf{h} \mid I(\mathbf{x}; \mathbf{y} \mid \mathbf{H}) < R \})$$

- ◇ At high SNR, $\mathbb{P}_e \geq$ probability of outage
- ◇ **Compound channel result:** The outage probability can be achieved with long block-length codes.
- ◇ Short block length **space time** codes - delay diversity, rotated QAM codes, space time trellis codes, space time turbo codes.

R and $\mathbb{P}_e$: a Coarser Scaling

◇ Coarser question: Zheng and Tse formulation

– **Rate** $= r \log(\text{SNR})$

– **Probability of error** $= \frac{1}{\text{SNR}^d}$

◇ Given r , find maximal $d = d^*(r)$

R and $\mathbb{P}_e$: a Coarser Scaling

- ◇ Coarser question: Zheng and Tse formulation
 - **Rate** $= r \log(\text{SNR})$
 - **Probability of error** $= \frac{1}{\text{SNR}^d}$
- ◇ Given r , find maximal $d = d^*(r)$
- ◇ A code with rate r is **tradeoff optimal for a channel** if the diversity gain over that channel is $d^*(r)$

Universal Tradeoff Optimality

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Universal Tradeoff Optimality

- ◇ A code is **universally** tradeoff optimal if it is tradeoff optimal for **every** channel distribution
- ◇ Compound channel formulation guarantees universal codes – with long enough block length (KW03)

Universal Tradeoff Optimality

- ◇ A code is **universally** tradeoff optimal if it is tradeoff optimal for **every** channel distribution
- ◇ Universal code design criterion (TV04)

$$\lambda_1 \lambda_2 \dots \lambda_{\min(n_r, n_t)} \geq \frac{\text{SNR}^{\min(n_r, n_t)}}{\text{SNR}^r}$$

- ◇ $\lambda_1 \leq \dots \leq \lambda_{n_t}$ are eigenvalues of $\mathbf{D}\mathbf{D}^\dagger$, $\mathbf{D} := \mathbf{X}(1) - \mathbf{X}(2)$
- ◇ Need this criterion for every pair of codewords

A Contrast

- ◇ Consider a MISO channel: $n_r = 1$
- ◇ Traditional design criterion for i.i.d. Rayleigh fading (TSC 98)

$$\text{maximize } \lambda_1 \lambda_2 \dots \lambda_{n_t} \quad (\text{determinant})$$

- ◇ Universal design criterion

$$\text{maximize } \lambda_1 \quad (\text{smallest singular value})$$

- have to protect against the worst case channel

Universal Codes

- ◇ Need rotational invariance

$$\mathbf{X}\mathbf{X}^* = \text{SNR} \quad \mathbf{I}$$

- full rate orthogonal designs are universal

Universal Codes

- ◇ Need **rotational invariance**

$$\mathbf{X}\mathbf{X}^* = \text{SNR} \quad \mathbf{I}$$

- full rate orthogonal designs are universal
- ◇ Nearly rotational invariant: **codes based on number theory**
 - Tilted QAM code (YW03, DV03), Golden code (BRV04), codes based on cyclic division algebra (Eli04)
 - Universally tradeoff optimal, but hard to decode

Universal Codes for the Parallel Channel

- ◇ A parallel channel is a MIMO channel with a specific transmit basis
- ◇ Simple **permutation** codes are universal
- ◇ For $L = 2$, **bit-reversal** is universal

Main Results

◇ Setting:

- coherent communication over short block length at high SNR
- universal tradeoff performance over a **restricted class** of channels

Main Result

◇ Setting:

- coherent communication over short block length at high SNR
- universal tradeoff performance over a **restricted class** of channels

◇ Main Result:

- Universal tradeoff optimal designs based on parallel channel codes
 - * engineering value: code is **robust** to channel modeling errors
 - * Simple encoding and decoding of **permutation codes**

MISO Channel Revisited

$$\mathbf{y}[m] = \mathbf{h}^* \mathbf{x}[m] + \mathbf{w}[m]$$

- ◇ Suppose $h_1, \dots, h_{n_t}$ are i.i.d.
 - no antenna is particularly vulnerable
- ◇ **Universality Result:**

It is tradeoff optimal to use one transmit antenna at a time

- ◇ Converts MISO channel into a parallel channel
 - can use the simple universal permutation codes

Restricted Class of Channels

$$\mathbf{H} = \mathbf{U}\mathbf{\Lambda}\mathbf{V}^*$$

- ◇ $\mathbf{V}^*$ is the Haar measure on $SU(n_t)$ and $\mathbf{\Lambda}$ independent of $\mathbf{V}$
- ◇ each transmit direction is equally likely
- ◇ i.i.d. Rayleigh fading belongs to this class

D-BLAST Architecture

- ◇ D-BLAST scheme for 2 transmit antennas:

$$X = \begin{bmatrix} 0 & a_2 & b_2 & c_2 \\ a_1 & b_1 & c_1 & 0 \end{bmatrix}$$

- ◇ Successive cancellation of streams
 - converts MIMO channel to a parallel channel
 - can use permutation codes
- ◇ Almost universally tradeoff optimal
 - initialization overhead

D-BLAST with ML decoding

$$X = \begin{bmatrix} 0 & a_2 & b_2 \\ a_1 & b_1 & 0 \end{bmatrix}$$

- ◇ Consider joint ML decoding of both streams
- ◇ **Claim:** Universal tradeoff performance same as that with successive cancellation
- ◇ Main result:

Universal tradeoff performance over the restricted class of channels

2×2 i.i.d. Rayleigh channel

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2×2 i.i.d. Rayleigh channel

Universality

- ◇ D-BLAST with joint ML decoding of two streams:

achieves the low rate part of the tradeoff curve over the restricted class of channels

- ◇ $\mathbf{H} = \mathbf{U}\mathbf{\Lambda}\mathbf{V}^*$

- $\mathbf{V}$ is independent of $\mathbf{\Lambda}$ and has the Haar measure
- Suppose $\lambda_1 \leq \lambda_2$ are the singular values

$$\mathbb{P} [\lambda_1^2 \leq \epsilon_1, \lambda_2^2 \leq \epsilon_2] \approx \epsilon_1^{k_1} \epsilon_2^{k_2}$$

with $k_1 \leq 1, k_2 = 3$.

3×2 i.i.d. Rayleigh Channel

◇ D-BLAST with two streams

$$X = \begin{bmatrix} 0 & 0 & a_3 & b_3 \\ 0 & a_2 & b_2 & 0 \\ a_1 & b_1 & 0 & 0 \end{bmatrix}$$

3×2 i.i.d. Rayleigh channel

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$n_t \times 2$ i.i.d. Rayleigh Channel

◇ D-BLAST with two streams

$$X = \begin{bmatrix} 0 & 0 & \dots & a_{n_t} & b_{n_t} \\ \vdots & \vdots & \vdots & \vdots & \vdots \\ 0 & a_2 & b_2 & \dots & 0 \\ a_1 & b_1 & 0 & \dots & 0 \end{bmatrix}$$

$n_t \times 2$ i.i.d. Rayleigh channel

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$n_t \times 2$ i.i.d. Rayleigh channel

V-Blast with ML decoding

- ◇ Send QAM independently across antennas (**space only code**)

2×2 i.i.d. Rayleigh fading

- ◇ space only V-Blast optimal for $r \geq 1$ on 2×2 Rayleigh channel

D-BLAST Time-Space Code

◇ D-BLAST **space-time** code:

$$X = \begin{bmatrix} 0 & a_2 & b_2 \\ a_1 & b_1 & 0 \end{bmatrix}$$

◇ D-BLAST **time-space** code:

$$X^* = \begin{bmatrix} 0 & a_1 \\ a_2 & b_1 \\ b_2 & 0 \end{bmatrix}$$

3×2 i.i.d. Rayleigh channel

D-BLAST Time-Space Code

◇ D-BLAST **space-time** and **time-space** codes:

$$X = \begin{bmatrix} 0 & 0 & \dots & a_{n_t} & b_{n_t} \\ \vdots & \vdots & \vdots & \vdots & \vdots \\ 0 & a_2 & b_2 & \dots & 0 \\ a_1 & b_1 & 0 & \dots & 0 \end{bmatrix} \quad X^* = \begin{bmatrix} 0 & \dots & 0 & a_1 \\ 0 & \dots & a_2 & b_1 \\ \dots & \dots & b_2 & 0 \\ a_{n_t} & \dots & \vdots & 0 \\ b_{n_t} & \vdots & 0 & 0 \end{bmatrix}$$

$n_t \times 2$ i.i.d. Rayleigh channel

Reprise

- ◇ Considered universality over a restricted class of MIMO channels
 - transmit angular basis is uniform
- ◇ Main result:

universal tradeoff optimality of D-BLAST with joint ML decoding

- ◇ Reference:

Chapter 9 of *Fundamentals of Wireless Communication*,
D. Tse and P. Viswanath, Cambridge University Press, 2005.