Multiple Antennas: A Network View

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MIMO in Wireless Networks

- Explosion of research in recent years
  - information theory
  - coding
  - signal processing

- Much focus on point-to-point channels

- To understand impact of multiple antennas in wireless networks, need broader view
Multiple Access Example

Question: what does adding one more antenna at each mobile buy me?
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• Looking at each point-to-point link in isolation:
Question: what does adding one more antenna at each mobile buy me?

- Looking at point-to-point link in isolation:
  - (roughly) doubles the link capacity.
• Looking at the network:
  – number of users is greater than number of receive antennas
  – increase in overall system capacity negligible

• But does adding that antenna still buy me something?
Outline of Talk

- Review of diversity-multiplexing tradeoff in point-to-point channels.
- Extension to multiple access scenario.
- Speculation on a theory for general networks.
Point-to-Point MIMO Channel

$M$ transmit and $N$ receive antennas.

I.I.D. Rayleigh fading model.
Degrees of Freedom

- point-to-point link: $M$ transmit, $N$ receive antennas
- i.i.d. Rayleigh fading (Foschini 96):

  \[ C \sim \min\{M, N\} \log \text{SNR} \quad \text{bits/s/Hz}. \]

- Multiple antennas provide $\min\{M, N\}$ degrees of freedom
- spatial multiplexing gain of $\min\{M, N\}$
- $C'$ is the ergodic capacity.
Diversity

- Ergodic capacity assumes infinite-depth interleaving
- Impossible in a slow fading environment
- Unreliability due to fading is a first-order issue.
- In 1 by 1 Rayleigh fading channel: very poor error probability.
- Example: for BPSK:

\[ P_e \sim \text{SNR}^{-1} \quad \text{at high SNR} \]

- In \( M \) by \( N \) channel, however,

\[ P_e \sim \text{SNR}^{-MN} \quad \text{at high SNR} \]

- Multiple antennas provide a maximum of \( MN \) diversity gain.
But each is only a single-dimensional view of the situation. The right way to formulate the problem is a tradeoff between the two types of gains.
**Fundamental Tradeoff**

Focus on high SNR and slow fading situation.

A space-time coding scheme of block length $T$ achieves

- **Spatial Multiplexing Gain** $r$ : if data rate $R = r \log \text{SNR}$ (bps/Hz)
- **Diversity Gain** $d$ : if error probability $P_e \sim \text{SNR}^{-d}$

Equivalently:

- $d \rightarrow r^{*}_{M,N}(d)$
- $r \rightarrow d^{*}_{M,N}(r)$

A tradeoff between data rate and error probability.
**Fundamental Tradeoff**

Focus on high SNR and slow fading situation. A space-time coding scheme of block length $T$ achieves

Spatial Multiplexing Gain $r$ :  if data rate $R = r \log \text{SNR}$ \,(bps/Hz) and

Diversity Gain $d$ :  if error probability $P_e \sim \text{SNR}^{-d}$

Fundamental tradeoff: for any $r$, the maximum diversity gain achievable: $d^*_{M,N}(r)$.

$$r \rightarrow d^*_{M,N}(r)$$

Equivalently:

$$d \rightarrow r^*_{M,N}(d)$$

A tradeoff between data rate and error probability.
Optimal Tradeoff

( Zheng, Tse 02) If block length \( T \geq M + N - 1 \):

Spatial Multiplexing Gain: \( r = \frac{R}{\log \text{SNR}} \)

Diversity Gain: \( d^*(r) \) \( \min\{M, N\}, 0 \) \( 0, MN \)

To guarantee a multiplexing gain of \( r \) \( \text{integer} \), the best diversity gain achievable for any space-time code is \( (M - r)(N - r) \).
(Zheng, Tse 02) If block length $T \geq M + N - 1$:

For multiplexing gain of $r$ ($r$ integer), best diversity gain achievable is $(M - r)(N - r)$. 

**Optimal Tradeoff**

Spatial Multiplexing Gain: $r = R / \log \text{SNR}$

Diversity Gain: $d^*(r)$

(min{M,N},0)

(0,MN)

(1,(M−1)(N−1))

(2, (M−2)(N−2))

(r, (M−r)(N−r))

(min{M,N},0)
Multiple Access

- For point-to-point, multiple antennas provide diversity and multiplexing gain.
- With $K$ users, multiple antennas discriminate signals from different users too.
- i.i.d. Rayleigh fading, $N$ receive, $M$ transmit antennas per user.
Multiuser Diversity-Multiplexing Tradeoff

Suppose we want every user to achieve an error probability:

\[ P_e \sim \text{SNR}^{-d} \]

and a data rate

\[ R = r \log \text{SNR} \text{ bits/s/Hz.} \]

What is the optimal tradeoff between 
\( d \) (diversity gain) and \( r \) (multiplexing gain)?

Assume a block length \( T \geq KM + N - 1 \).
Optimal Multiuser D-M Tradeoff

- For $r = 0$, diversity is $MN$
- For $r = \min\{M, \frac{N}{K}\}$, diversity is 0
Multiuser Tradeoff: \( M < N/(K + 1) \)

- diversity-multiplexing tradeoff of each user is \( d^*_{M,N}(r) \)
- as though it is the only user in the system
Multiuser Tradeoff: \( M > N/(K + 1) \)

- \( r \leq N/(K + 1) \): Single-user tradeoff curve
Multiuser Tradeoff: \( M > N/(K + 1) \)

- \( r \leq N/(K + 1) \): Single-user tradeoff curve
- \( r \) from \( N/(K + 1) \) to \( \min\{M, N/K\} \):
  - tradeoff as though the \( K \) users are pooled together: \( KM \) antennas and rate \( Kr \),
Question: what does adding one more antenna at each mobile buy me?
Scenario of 1 transmit antenna

Adding one more transmit antenna does not increase the number of degrees of freedom for each user. However, it increases the maximum diversity gain from $N$ to $2N$. More generally, it improves the diversity gain $d(r)$ for every $r$.

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**Spatial Multiplexing Gain:** $r = R / \log \text{SNR}$

**Diversity Gain:** $d(r)$

**Optimal tradeoff**
**Answer: Adding one more transmit antenna**

- No increase in number of degrees of freedom
- However, increases the maximum diversity gain from $N$ to $2N$.
- Improves diversity gain $d(r)$ for every $r$. 
Tradeoff Between Users

• We have been looking at the symmetrical, equal rate case.

• More generally, we can ask:

  What is the optimal tradeoff between the achievable multiplexing
  gains for a given diversity gain $d$?

• Given by the multiplexing gain region $C(d)$ for a given $d$. 
• Multiplexing gain region $\mathcal{C}(d)$ is a cube: $r_i \leq r^*_{M,N}(d)$

• Single user performance for every user

• Require:
  - $M \leq N/(K + 1)$ (large number of receive antennas), or
  - $M > N/(K + 1)$ but $d \geq d^*_{K_M,N}[N/(K + 1)]$ (high diversity requirement)
Multiplexing Region: General Case

If \( d \in \left[ d_{(k-1)M,N}[N/k], d_{kM,N}[N/(k + 1)] \right] \):

\[
C(d) = \left\{ (r_1, \ldots, r_K): \sum_{i \in S} r_i < r_{|S|M,N}(d), \quad \forall S \text{ with } |S| = 1 \text{ or } |S| \geq k \right\}
\]

- \( r_{|S|M,N}(d) \) is point-to-point M-D tradeoff with \(|S| \) Tx and \( N \) Rx antennas.
- As \( d \) decreases, more and more constraints become active.
- Finally, \( 2^K - 1 \) constraints are active: \( C(d) \) is a polymatroid.
$r_{2M,N}^*(d)$ is total multiplexing gain in system with $2M$ transmit antennas pooled together.
Suboptimal Receiver: the Decorrelator/Nuller

- Consider case of $M = 1$ transmit antenna for each user
- Number of users $K < N$
Tradeoff for the Decorrelator

- Maximum diversity gain is $N - K + 1$
- “costs $K - 1$ diversity to null out $K - 1$ interferers” (Winters et al '93)
Tradeoff for the Decorrelator

- Maximum diversity gain is $N - K + 1$
- “costs $K - 1$ diversity to null out $K - 1$ interferers” (Winters et al ’93)
- Adding one receive antenna provides:
  - either more reliability per user
  - or accommodate 1 more user at the same reliability.
Tradeoff for the Decorrelator

- Optimal tradeoff curve also a straight line
  - but with a maximum diversity gain of $N$.
- Adding one receive antenna provides more reliability per user and accommodate 1 more user.
Multiple Antennas in General Networks

Multiple antennas serve multiple functions:

- diversity
- spatial multiplexing
- multiple access
- broadcast
- interference suppression
- cooperative relaying (distributed antennas)
- etc ....

What is the fundamental performance tradeoff in general?

Our approach may give a simple picture.