

ECE544
Statistical Image and Video Processing

Homework #3

Due October 15, 2008

Problem 1 (*Hidden Markov Model*). Consider the following HMM. The hidden sequence S_0, S_1, S_2, S_3 is a homogeneous Markov chain with alphabet $\Lambda = \{0, 1\}$ and transition probability matrix $Q = \begin{pmatrix} 0.9 & 0.1 \\ 0.1 & 0.9 \end{pmatrix}$. The observed variables are $\mathbf{X} = \{X_0, \dots, X_7\} \in \Lambda^8$. The conditional pmf for \mathbf{X} given \mathbf{S} is given by

$$p(\mathbf{x}|\mathbf{s}) = \prod_{i=0}^3 Q(x_{2i}|s_i)Q(x_{2i+1}|s_i).$$

Give a computationally efficient algorithm for evaluating $p(x_7|s_0)$.

Problem 2 (*Simplified graphical model for hand silhouettes*). Say the global orientation of the hand is represented by a variable O , all fingers have prespecified lengths, and the angle subtended by fingers 1 and 5 (the pinkie and the thumb) is denoted by A . The orientation of the five fingers is represented by angular variables F_1, \dots, F_5 . In this simplified model, the hand is parameterized by the 7 variables F_1, \dots, F_5, O, A .

These variables are modeled as random with the following joint pdf:

$$p(f_1, \dots, f_5, o, a) = \frac{1}{Z} \psi_{15}(f_1, f_5|o, a) \psi_{12}(f_1, f_2) \psi_{23}(f_2, f_3) \psi_{34}(f_3, f_4) \psi_{45}(f_4, f_5) p(o) p(a).$$

1. Draw a graphical representation of this model, and give a computationally efficient algorithm for evaluating $p(f_3)$.
2. Does this model appear physiologically reasonable?
Hint: to think about this question, lay one hand flat on a table, hold the pinkie and the thumb firm with the other hand, and try to move the three free fingers one at a time.
3. Assume now that $O = o^*$ and $A = 10$ (degrees) with probability 1, and that

$$\begin{aligned} \psi_{i,i+1}(f_i, f_{i+1}) &= \exp\{f_i - f_{i+1}\} 1_{\{f_i < f_{i+1}\}} : i = 1, 2, 3 \\ &= \exp\{\frac{1}{4}(f_i - f_{i+1})\} 1_{\{f_i < f_{i+1}\}} : i = 4 \\ \psi_{15}(f_1, f_5|o^*, 10) &= \exp\{10^6|f_1 + 5| + 10^6|f_5 - 5|\}. \end{aligned}$$

Write a computer program and evaluate the expected value of F_3 .

Hint: you should be able to make a rough guess of that value before writing the program.