

ECE544
Statistical Image and Video Processing

Homework #2
Due October 1, 2008

Problem 1 (*Computer Experiment on AR Processes*)

- (a) Compute the mean and the first 4×4 covariance samples $R_X(n)$, $0 \leq n_1, n_2 \leq 3$ for the five images (tif format) on the Web site. This will be done by computing spatial averages and making a stationarity assumption. Discuss the validity of fits of the form $R_X(n) = C\rho^{|n_1|+|n_2|}$ and $R_X(n) = C\rho^{\|n\|}$.
- (b) Describe a simple and computationally efficient algorithm for generating realizations of the following process: $x(n)$ is stationary and Gaussian distributed with mean 100 and covariance $R_X(n) = 30 * \rho^{|n_1|+|n_2|}$.
- (c) Generate a realization of the process in (b), using the following values for ρ : .1, .5 and .95.

Problem 2 (*Binary Gibbs Random Fields*) Propose an energy function that strongly favors configurations made of nearest-neighbor pairs of black pixels surrounded by white pixels, or vice-versa.

Problem 3 (*Computer Experiment on Ising Model*) Use Gibbs sampling to generate realizations from the Ising model

$$P(x) = Z^{-1} e^{-\frac{1}{T}U(x)}, \quad U(x) = \sum_{i \sim j} x_i x_j$$

(where each $x_i = \pm 1$, and the sum is over nearest neighbors).

Use a periodic 50×50 lattice and equiprobable, i.i.d binary random variables as your initial distribution. Use the following values for T^{-1} : .1, .6, .88, 1.5 and 10. Observe the apparent convergence of the distribution to a steady state as the number of iterations increases. Explain the aspect of typical realizations for different values of T upon convergence.

Problem 4 (Optional). Repeat Problem 3 with the 8-level Potts model.

Problem 5 (*Maximum-Likelihood (ML) Parameter Estimation*). Consider the GGMRF model with scale parameter σ and exponent p . The energy function takes the form

$$U(x) = \sum_i x_i^2 + \sum_{i \sim j} \left| \frac{x_i - x_j}{\sigma} \right|^p,$$

where $0 < p \leq 2$, and the second sum is over all nearest neighbors.

1. Explain why exact ML estimation of σ and p is difficult.
2. Develop an approximate ML estimation method for σ and p and apply it to the five tif images on the Web page. Comment.