

ECE544
Statistical Image and Video Processing

Homework #1

Due TUESDAY September 23, 2008

Problem 1 (*Object Randomly Thrown on the Floor*). Let $\psi(t)$ be a deterministic signal defined over the square $[0, 1]^2$, and extended outside the boundaries of the square using periodic extensions. Consider the random process $x(t) = \psi(t - d)$, where d is a random location parameter, uniformly distributed over the unit square.

- (a) Give expressions for the mean and covariance of the process $x(t)$. Is $x(t)$ WSS?
- (b) Is $x(t)$ ergodic in the mean? ergodic in correlation?
- (c) Is $x(t)$ strictly stationary?
- (d) Repeat (a)–(c) when the distribution of the location parameter d is nonuniform.
- (e) Repeat (d) when $\psi(t)$ is a strictly stationary periodic random process (with period equal to the unit square), and d is independent of $\psi(t)$.

Problem 2. Let $x(t)$ be a stationary random process in the plane, and define $y(t) = x(R_\theta t)$ where R_θ is a 2×2 rotation matrix with rotation angle θ uniformly distributed over $[0, 2\pi)$.

- (a) (8 pts) Prove that $y(t)$ is stationary and isotropic.
- (b) (7 pts) Prove that $z(t) = T[y(t)]$ is stationary and isotropic, where $T[\cdot]$ is an arbitrary point operation.

Problem 3. Formulate a statistical model for the 9 images in the Lecture Notes. A detailed statistical model is not requested here, but you are asked to indicate whether the random process that has generated those images is stationary, isotropic, and/or ergodic.