Problem 1 (Object Randomly Thrown on the Floor). Let $\psi(t)$ be a deterministic signal defined over the square $[0, 1]^2$, and extended outside the boundaries of the square using periodic extensions. Consider the random process $x(t) = \psi(t - d)$, where $d$ is a random location parameter, uniformly distributed over the unit square.

(a) Give expressions for the mean and covariance of the process $x(t)$. Is $x(t)$ WSS?

(b) Is $x(t)$ ergodic in the mean? ergodic in correlation?

(c) Is $x(t)$ strictly stationary?

(d) Repeat (a)–(c) when the distribution of the location parameter $d$ is nonuniform.

(e) Repeat (d) when $\psi(t)$ is a strictly stationary periodic random process (with period equal to the unit square), and $d$ is independent of $\psi(t)$.

Problem 2. Let $x(t)$ be a stationary random process in the plane, and define $y(t) = x(R_\theta t)$ where $R_\theta$ is a $2 \times 2$ rotation matrix with rotation angle $\theta$ uniformly distributed over $[0, 2\pi)$.

(a) (8 pts) Prove that $y(t)$ is stationary and isotropic.

(b) (7 pts) Prove that $z(t) = T[y(t)]$ is stationary and isotropic, where $T[\cdot]$ is an arbitrary point operation.

Problem 3. Formulate a statistical model for the 9 images in the Lecture Notes. A detailed statistical model is not requested here, but you are asked to indicate whether the random process that has generated those images is stationary, isotropic, and/or ergodic.