RFIT: a new algorithm for matrix rank minimization

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***Abstract*—In this paper we introduce a novel algorithm for solving the low rank matrix recovery problem. The algorithm is called Residual Feedback Iterative Thresholding (RFIT) and introduces a novel choice of update direction inspired by the Approximate Message Passing (AMP) compressed sensing solver. Empirical results suggest that RFIT requires fewer measurements than several state of the art algorithms for accurate reconstruction.**

I. Introduction and Prior Art

The low rank matrix recovery problem has come under intense study over the past five years. A popular technique for matrix recovery is called iterative thresholding and takes the following form. At each step one chooses a direction G^k away from the current estimate X^k of the unknown low rank matrix M. One then applies an operator to $X^k + \tau_k G^k$ that enforces low rankness. For example the algorithms of [1] and [2] use singular value thresholding. G^k is typically minus the gradient of the error term $\|\mathcal{A}(X^k) - b\|_2^2$, i.e $G^k = \mathcal{A}^*\left(b - \mathcal{A}\left(X^k\right)\right)$. In this paper, we introduce a novel choice of G^k that improves convergence speed and reconstruction error.

II. The RFIT Algorithm

RFIT is inspired by approximate message passing (AMP), a sparse vector recovery solver [3]. From an optimization point of view, AMP is a simple elementwise soft thresholding procedure with a unique choice of G^k. Whereas most iterative schemes choose G^k to be minus the gradient at the current X^k, AMP icludes an additional feedback term.

Considering the performance of AMP, we wanted to find out if residual (or gradient) feedback could help in the related, but different problem of low rank matrix recovery. Our empirical results suggest that it can (see figures 1 and 2).

Algorithm 1 Residual Feedback Iterative Thresholding (RFIT)

Inputs: $\mathcal{A}, b, \beta, \tau, p =$ estimated rank of M
1. **Initialize** $X^0 = 0, r^0 = 0$ $k = 1$
2. If the stopping criterion is met then *return*
3. $r^k = \beta r^{k-1} + b - \mathcal{A}(X^k)$
4. X^{k+1} = truncate $(X^k + \tau\mathcal{A}^*(r^k))$ to first p singular values
5. $k = k + 1$, loop to 2

Output: X^k

RFIT looks superficially similar to SVP [1], however the inclusion of a residual feedback term βr^{k-1} adds an important momentum effect. Indeed RFIT is the same as the heavy-ball method [4], except for the thresholding peformed at each iteration. After the rank of $(X^k + \tau\mathcal{A}^*(r^k))$ converges to p, the iterations are identical to heavy-ball. In unconstrained optimization, the heavy-ball method improves the convergence rate

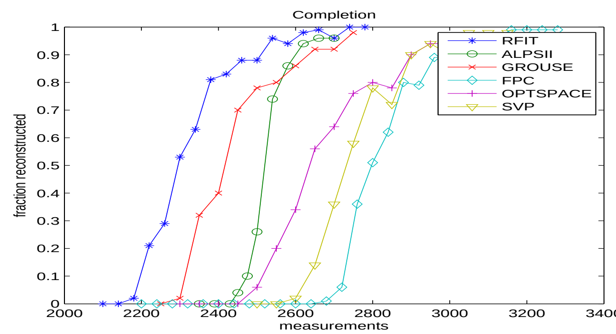

Fig. 1: Completion: RFIT in blue. M is 100×100 matrix of rank 10 with uniformly chosen revealed entries. Fraction rel-error < 0.001 (success) over 50 trials v # measurements.

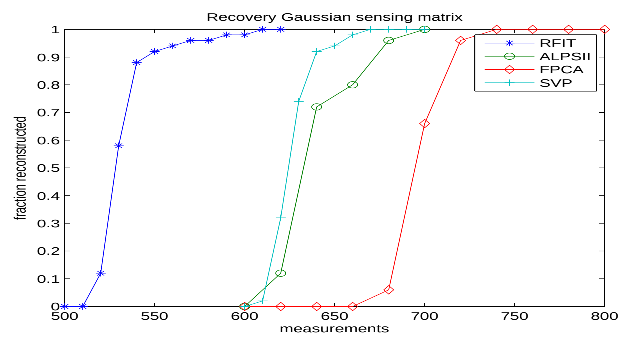

Fig. 2: Recovery: RFIT is blue. Gaussian measurement matrix. M is 50×50 rank 5. Fraction success v # measurements.

of standard gradient descent esp. with ill-conditioned Hessian. In figures 1 and 2 we compare RFIT with algorithms from ([5], [1], [2], [6], [7]). We empirically chose $\beta = 0.85$, $\tau = 2$ for completion and $\tau = 0.5$ for Gaussian measurements. Deriving optimal β and τ is an open question. The combination of heavy-ball with thresholding needs to be analyzed.

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