University of Illinois
ECE 598

Spring 2010
Prof. Douglas L. Jones

# Practice Final Exam

May 12, 2010

| Problem | Pts. | Score |
|---|---|---|
| 1 | 15 | |
| 2 | 15 | |
| 3 | 10 | |
| 4 | 20 | |
| 5 | 4 | |
| 6 | 10 | |
| 7 | 20 | |
| 8 | 10 | |
| 9 | 10 | |
| 10 | 15 | |
| Total | 100 | |

**Name** ________________________________

**Score** ____________

**Please do not turn this page over until told to do so.**

You may not use any books, calculators, or notes other than three <u>handwritten</u> two-sided note sheets of 8.5" x 11" paper.

**(15 Pts.)**

1. A periodic signal $x(t)$ with a period $T = 4$ sec has trigonometric Fourier series coefficients $a_0 = 5$, $a_1 = 3$, $b_3 = 2$; all others are equal to zero. Give the signal $x(t)$.

**(15 Pts.)**

2. Calculate the Fourier transform of the signal $x(t) = u(t) - u(t-2) + \delta(t-3)$.

**(10 Pts.)**

3. The length-4 sequences $x_1[n]$ and $x_2[n]$ have the DFTs $X_1 = \{1, 2j, 1, -1\}$ and $X_2 = \{1, 2, 1, 1\}$, respectively.

   (a) What is the DFT of $2x_1[n] + x_2[n]$?

   (b) What is the DFT of $e^{j\pi n}x_2[n]$?

   (c) What is $\sum_{n=0}^{3} |x_1[n]|^2$?

   (d) What is $x_1[0]$?

   (e) What is $x_2[1]$?

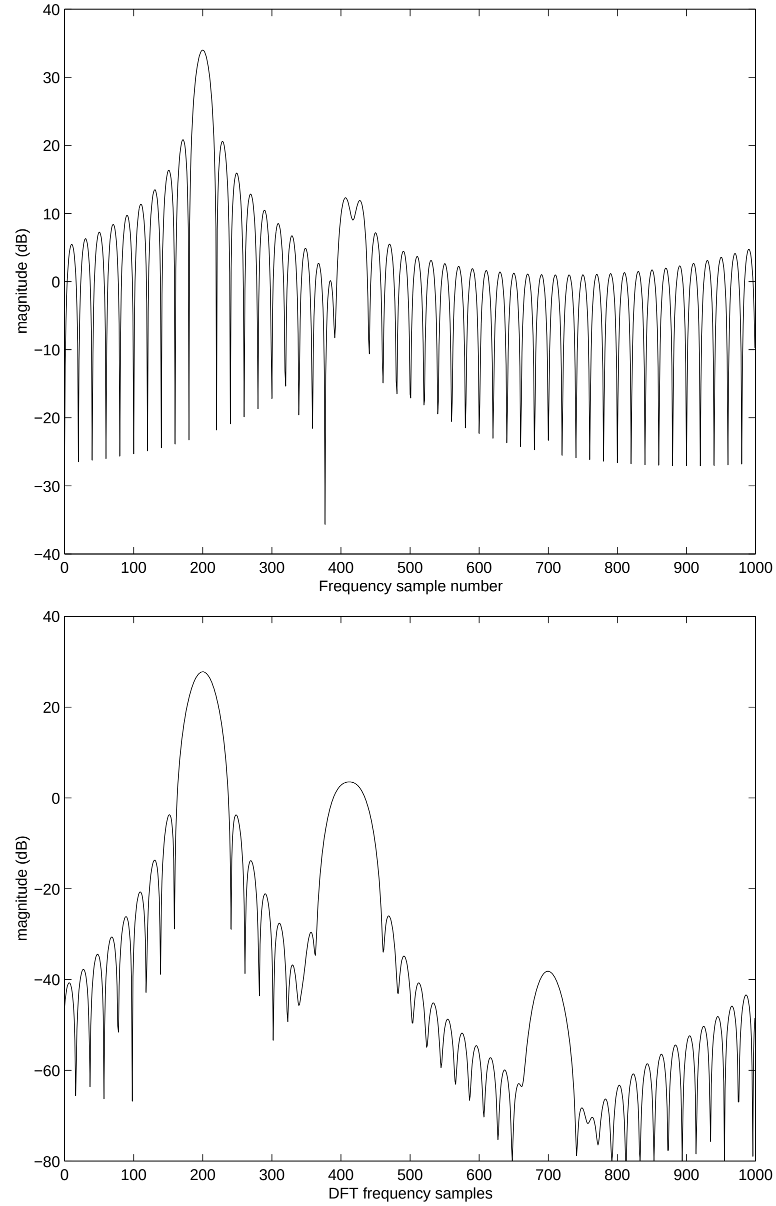
**(20 Pts.)**

4. The two figures on the next page show the magnitude (in dB) of the length-1000 zero-padded DFT of 50 samples of an analog signal bandlimited to 5 kHz, sampled at a rate of $f_s = 10$ kHz. The first figure was made with a rectangular window, and the second was made with a Hann window.

   (a) Is the sequence $x[n]$ (a) real-valued, (b) purely imaginary, (c) complex-valued, (d) not enough information to tell? (Circle one)

   (b) How many frequency components can you be sure are present? At what DFT indices $m$ are they at?

   (c) For each of these indices, tell what DTFT frequency $\omega$ and also what analog frequency $f$ (in Hz) it corresponds to.


40
30
20
10
0
−10
−20
−30
−40
magnitude (dB)
0 100 200 300 400 500 600 700 800 900 1000
Frequency sample number
40
20
0
−20
−40
−60
−80
magnitude (dB)
0 100 200 300 400 500 600 700 800 900 1000
DFT frequency samples

**(4 Pts.)**

5. The output, $y[n]$, of a discrete-time system is related to its input, $x[n]$, by

$$y[n] = x[-n + 10]$$

Determine whether the system possesses the following properties ($+1$ for correct answer, $-1$ for wrong answer; min 0, max 4 points):

(a) Linear Y/N

(b) Shift-invariant Y/N

(c) Causal Y/N

(d) BIBO stable Y/N

**(10 Pts.)**

6. We wish to bandpass filter an analog signal to keep the frequencies between 2 kHz and 3 kHz and remove all other frequencies. The signal is bandlimited to 4 kHz.

(a) What is the minimum sampling frequency required to prevent aliasing of the analog signal?

(b) Suppose the sampling rate to be used is $f_s = 10$ kHz. The signal will be processed with an ideal DSP system (ideal A/D, digital filter, and D/A). Plot below the frequency response of the required digital filter.

**(10 Pts.)**

7. The analog input to an ideal DSP system with a sampling period $T = 1/1000$ is $x_a(t) = 3 + 2\cos(100\pi t) - \sin(200\pi t) + \sin(800\pi t) + cos(1400\pi t)$. The frequency response of the digital filter is $1 + |\omega|$ for $-\pi \le \omega < \pi$. What is the output, $y_a(t)$?

**(20 Pts.)**

8. Consider the transfer function $H(z) = \frac{z+1}{z-0.8}$.

   (a) Does this transfer function correspond to an IIR or FIR filter?

   (b) Find and plot the poles and zeros.

   (c) Find the difference equation, $y[n] = \ldots$, for this system.

   (d) Is this a BIBO stable filter?

   (e) Is it a lowpass, highpass, bandpass, or bandstop filter?

   (f) Is it a linear-phase filter?

   (g) Find the frequency response, $H(\omega)$.

   (h) Sketch the magnitude of the frequency response.

**(10 Pts.)**

9. Consider the FIR filter $[1 \quad -1]$ ($h[n] = \delta[n] - \delta[n-1]$).
   (a) Calculate the frequency response, $H(\omega)$, of this filter.
   (b) Is it a generalized-linear-phase filter?
   (c) Is it a lowpass, highpass, bandpass, or bandstop filter?

**(15 Pts.)**

10. Design a length-5 generalized linear phase FIR lowpass filter with a cutoff frequency of $\omega_c = \pi/2$ using the window design method with a Hamming window. Give the filter coefficients $h[n]$ as your answer.