Practice Final Exam

May 12, 2010

Name ________________________________

Score __________

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Please do not turn this page over until told to do so.

You may not use any books, calculators, or notes other than three handwritten two-sided note sheets of 8.5” x 11” paper.
1. A periodic signal $x(t)$ with a period $T = 4$ sec has trigonometric Fourier series coefficients $a_0 = 5, a_1 = 3, b_3 = 2$; all others are equal to zero. Give the signal $x(t)$.

2. Calculate the Fourier transform of the signal $x(t) = u(t) - u(t - 2) + \delta(t - 3)$. 
3. The length-4 sequences \( x_1[n] \) and \( x_2[n] \) have the DFTs \( X_1 = \{1, 2j, 1, -1\} \) and \( X_2 = \{1, 2, 1, 1\} \), respectively.

(a) What is the DFT of \( 2x_1[n] + x_2[n] \)?

(b) What is the DFT of \( e^{j\pi n}x_2[n] \)?

(c) What is \( \sum_{n=0}^{3} |x_1[n]|^2 \)?

(d) What is \( x_1[0] \)?

(e) What is \( x_2[1] \)?
4. The two figures on the next page show the magnitude (in dB) of the length-1000 zero-padded DFT of 50 samples of an analog signal bandlimited to 5 kHz, sampled at a rate of $f_s = 10$ kHz. The first figure was made with a rectangular window, and the second was made with a Hann window.

(a) Is the sequence $x[n]$ (a) real-valued, (b) purely imaginary, (c) complex-valued, (d) not enough information to tell? (Circle one)

(b) How many frequency components can you be sure are present? At what DFT indices $m$ are they at?

(c) For each of these indices, tell what DTFT frequency $\omega$ and also what analog frequency $f$ (in Hz) it corresponds to.
5. The output, \( y[n] \), of a discrete-time system is related to its input, \( x[n] \), by

\[ y[n] = x[-n + 10] \]

Determine whether the system possesses the following properties (+1 for correct answer, −1 for wrong answer; min 0, max 4 points):

(a) Linear
(b) Shift-invariant
(c) Causal
(d) BIBO stable

(10 Pts.)

6. We wish to bandpass filter an analog signal to keep the frequencies between 2 kHz and 3 kHz and remove all other frequencies. The signal is bandlimited to 4 kHz.
(a) What is the minimum sampling frequency required to prevent aliasing of the analog signal?

(b) Suppose the sampling rate to be used is $f_s = 10$ kHz. The signal will be processed with an ideal DSP system (ideal A/D, digital filter, and D/A). Plot below the frequency response of the required digital filter.
(10 Pts.)

7. The analog input to an ideal DSP system with a sampling period \( T = 1/1000 \) is \( x_a(t) = 3 + 2 \cos(100\pi t) - \sin(200\pi t) + \sin(800\pi t) + \cos(1400\pi t) \). The frequency response of the digital filter is \( 1 + |\omega| \) for \(-\pi \leq \omega < \pi\). What is the output, \( y_a(t) \)?

(20 Pts.)

8. Consider the transfer function \( H(z) = \frac{z+1}{z-0.8} \).
   (a) Does this transfer function correspond to an IIR or FIR filter?
   (b) Find and plot the poles and zeros.
   (c) Find the difference equation, \( y[n] = \ldots \), for this system.
   (d) Is this a BIBO stable filter?
   (e) Is it a lowpass, highpass, bandpass, or bandstop filter?
   (f) Is it a linear-phase filter?
   (g) Find the frequency response, \( H(\omega) \).
   (h) Sketch the magnitude of the frequency response.
9. Consider the FIR filter \([1 - 1]\) \((h[n] = \delta[n] - \delta[n - 1])\).

(a) Calculate the frequency response, \(H(\omega)\), of this filter.
(b) Is it a generalized-linear-phase filter?
(c) Is it a lowpass, highpass, bandpass, or bandstop filter?

10. Design a length-5 generalized linear phase FIR lowpass filter with a cutoff frequency of \(\omega_c = \pi/2\) using the window design method with a Hamming window. Give the filter coefficients \(h[n]\) as your answer.