Homework # 5
Due May 5, 2010

1. Consider the difference equation $y[n] = -0.81y[n-2] + 0.5x[n] - 0.5x[n-2]$.
   (a) Does this difference equation implement an IIR or FIR filter?
   (b) Compute samples of the impulse response $h[n]$ for $n = -1, 0, 1, 2, 3$ by hand by applying the input $x[n] = \delta[n]$.
   (c) Find the transfer function, $H(z) = Y(z)/X(z)$, of this system.
   (d) Find and plot the poles and zeros.
   (e) Is this a BIBO stable filter? [Hint: check the poles.]
   (f) Is it a lowpass, highpass, bandpass, or bandstop filter?
   (g) Is it a linear-phase filter?
   (h) Find the frequency response, $H(\omega)$.
   (i) Sketch the magnitude of the frequency response.

2. Consider the FIR filter $[1\ \ 2\ \ 1]$ ($h[n] = \delta[n] + 2\delta[n-1] + \delta[n-2]$).
   (a) Calculate the frequency response, $H(\omega)$, of this filter.
   (b) Is it a linear-phase filter?
   (c) Is it a lowpass, highpass, bandpass, or bandstop filter?
   (d) Calculate and sketch the magnitude and phase of this filter.

3. Tell whether the following filters are symmetric, antisymmetric, or neither, and whether or not they are generalized linear phase filters.
   (a) $[-1\ \ 3\ \ 3\ \ -1]$
   (b) $[-1\ \ 3\ \ 3\ \ 1]$
   (c) $[-1\ \ -3\ \ 3\ \ 1]$
   (d) $[-1\ \ -1\ \ 3\ \ 3\ \ -1]$

4. Design a length-5 generalized linear phase FIR highpass filter with a cutoff frequency of $\omega_c = 2\pi/3$ using the window design method with a Hamming window. Give the filter coefficients as your answer. Either show your work (hand calculations) or your matlab code that computes the filter coefficients. Plot the magnitude and phase from $-\pi < \omega < \pi$ using Matlab. [Hint: `fftshift(abs(fft(h,1000)))` will give you 1000 samples of the magnitude over $-\pi < \omega < \pi$.]