Homework # 4
Due April 14, 2010

1. Consider the signal $x(t) = 3 + \cos(400\pi t) + 2\sin(800\pi t) + \cos(1000\pi t)$.
   (a) Determine and list all of the analog frequencies in the signal $x(t)$.
   (b) What is the minimum sampling frequency f_s to avoid aliasing?
   (c) What is the corresponding Nyquist frequency for this sampling rate/frequency?
   (d) For a sampling frequency of $f_s = 2000$ Hz, find $x[n]$.
   (e) What discrete-time frequencies ω, $-\pi < \omega \leq \pi$ will the analog frequencies be mapped to for this sampling rate?
   (f) If we take a length-800 DFT of 800 samples of this signal, for which k's will the DFT samples $X[k]$ be non-zero, and to which DTFT and analog frequencies do they correspond? (You might want to check your answer using Matlab.)

2. Say that we wish to digitally bandpass filter a signal bandlimited to 1 kHz to pass the frequencies between 200 Hz $< |\omega| <$ 300 Hz and to remove all other frequencies.
   (a) What is the minimum sampling frequency f_s needed to avoid aliasing?
   (b) For this frequency, what are the corresponding discrete-time frequencies ω_l and ω_h corresponding to the lower and upper cutoff frequencies for the bandpass filter?
   (c) Sketch the frequency response $|H(\omega)|$ of this filter.
   (d) Suppose instead a sampling frequency of 10 kHz is used. Now what are the necessary lower and higher cutoff frequencies ω_l and ω_h, respectively, of the digital filter?

3. Consider the DFT shown in the last problem of Homework 3. Assuming that the data samples came from an original complex-valued analog signal bandlimited to frequencies between -5 kHz $< f <$ 5 kHz sampled at a sampling rate $f_s = 10$ kHz, determine the analog frequency of all of the five frequency components.

4. Determine whether or not the following systems are linear, shift invariant, caual, and BIBO stable
   (a) $y[n] = x[n] - x[0]$
   (b) $y[n] = \log(\sum_{k=0}^{\infty} x[n-k])$

5. Compute the convolution $y[n] = x[n] * h[n]$ for $x[n] = [\ 1\ \ 2\ \ 0\ \ 2\ ]$ (that is, $x[n] = \delta[n] + 2\delta[n-1] + 2\delta[n-3]$) and $h[n] = [\ 1\ \ -1]$ (that is, $h[n] = \delta[n] - \delta[n-1]$) by hand. Then check your answer using Matlab using the "conv" function!