Homework # 3
Due March 10, 2010

1. Calculate the discrete-time Fourier transform (DTFT) $X_1(\omega)$ of the signal

$$x_1[n] = \begin{cases} 1, & -5 \leq n \leq 5 \\ 0, & \text{elsewhere} \end{cases}$$

Simplify your expression to a real-valued function of ω.

2. Write the above signal $x_1[n]$ in terms of step functions $u[n-m]$. (Hint: turn it on at the appropriate time m and off again at the appropriate time by subtracting off another step function.)

3. Use the time-shift property to find the DTFT $X_3(\omega)$ of the signal

$$x_3[n] = \begin{cases} 1, & 0 \leq n \leq 10 \\ 0, & \text{elsewhere} \end{cases}$$

using your result above.

4. Use the modulation property to find the DTFT $X_4(\omega)$ of the signal

$$x_4[n] = \begin{cases} \cos\left(\frac{\pi}{2}n\right), & -5 \leq n \leq 5 \\ 0, & \text{elsewhere} \end{cases}$$

using your result above. (Hint: use Euler's relations to express the cosine in terms of complex exponentials, then apply the modulation property.)

5. Show that the inverse DTFT of $X_5(\omega) = 2\pi\delta(\omega - \pi)$ for $-\pi < \omega \leq \pi$ is $x_5[n] = (-1)^n$ (that is, $x_5[n] = \ldots \quad 1 \; -1 \; 1 \; -1 \; 1 \; -1 \quad \ldots$).

6. Compute the length-4 DFT of $x[n] = [\,1 \;\; 2 \;\; 0 \;\; 2\,]$ by hand. Then check your answer using Matlab!

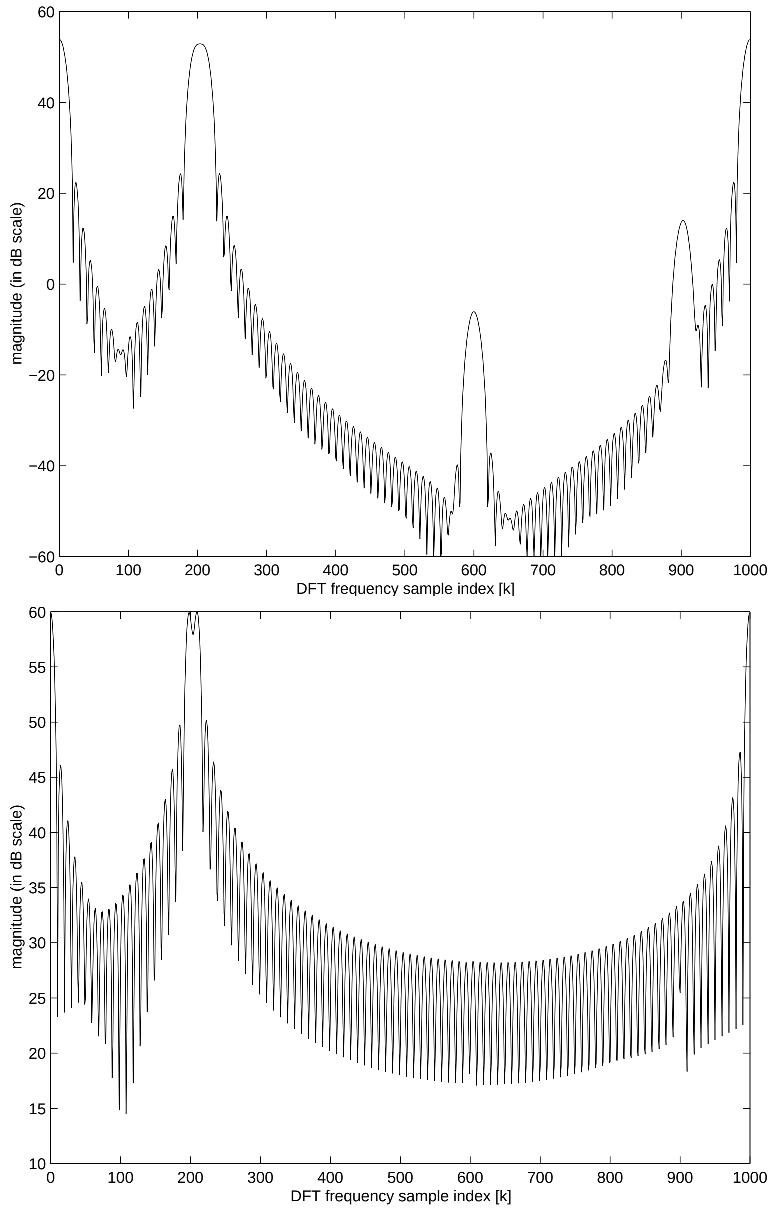
7. The two figures on the next page show the magnitude (in dB) of the length-1000 zero-padded DFT of 100 samples of a discrete-time signal composed of several complex-valued sinusoids $x[n] = A_1 \exp(j\omega_1 n) + A_2 \exp(j\omega_2 n) + \ldots$. One figure was made with a rectangular ("boxcar") window, and the other was made with a Hann window.

   (a) Which figure (the **first** or the **second**) was made with the rectangular window?

   (b) Is the sequence $x[n]$ **real-valued**, **purely imaginary**, **complex-valued**, or is there **not enough information to tell**? Explain your answer to receive credit.

   (c) How many frequency components can you be sure are present? For each of these frequency components, give the corresponding DTFT frequency ω_i, $-\pi \leq \omega_i < \pi$.

   (d) For these signal components, give the corresponding magnitudes $|A_i|$. (Hint: What is $X[0]$?)


60
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magnitude (in dB scale)
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DFT frequency sample index [k]
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magnitude (in dB scale)
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DFT frequency sample index [k]