1. Calculate the discrete-time Fourier transform (DTFT) \( X_1(\omega) \) of the signal

\[
x_1[n] = \begin{cases} 
1, & -5 \leq n \leq 5 \\
0, & \text{elsewhere}
\end{cases}
\]

Simplify your expression to a real-valued function of \( \omega \).

2. Write the above signal \( x_1[n] \) in terms of step functions \( u[n-m] \). (Hint: turn it on at the appropriate time \( m \) and off again at the appropriate time by subtracting off another step function.)

3. Use the time-shift property to find the DTFT \( X_3(\omega) \) of the signal

\[
x_3[n] = \begin{cases} 
1, & 0 \leq n \leq 10 \\
0, & \text{elsewhere}
\end{cases}
\]

using your result above.

4. Use the modulation property to find the DTFT \( X_4(\omega) \) of the signal

\[
x_4[n] = \begin{cases} 
\cos\left(\frac{\pi}{2}n\right), & -5 \leq n \leq 5 \\
0, & \text{elsewhere}
\end{cases}
\]

using your result above. (Hint: use Euler’s relations to express the cosine in terms of complex exponentials, then apply the modulation property.)

5. Show that the inverse DTFT of \( X_5(\omega) = 2\pi\delta(\omega - \pi) \) for \(-\pi < \omega \leq \pi\) is \( x_5[n] = (-1)^n \) (that is, \( x_5[n] = \ldots 1 - 1 1 - 1 1 - 1 \ldots \)).

6. Compute the length-4 DFT of \( x[n] = [1 \ 2 \ 0 \ 2] \) by hand. Then check your answer using Matlab!

7. The two figures on the next page show the magnitude (in dB) of the length-1000 zero-padded DFT of 100 samples of a discrete-time signal composed of several complex-valued sinusoids \( x[n] = A_1 \exp(j\omega_1 n) + A_2 \exp(j\omega_2 n) + \ldots \). One figure was made with a rectangular (“boxcar”) window, and the other was made with a Hann window.

(a) Which figure (the first or the second) was made with the rectangular window?

(b) Is the sequence \( x[n] \) real-valued, purely imaginary, complex-valued, or is there not enough information to tell? Explain your answer to receive credit.

(c) How many frequency components can you be sure are present? For each of these frequency components, give the corresponding DTFT frequency \( \omega_i \), \(-\pi \leq \omega_i < \pi\).

(d) For these signal components, give the corresponding magnitudes \( |A_i| \). (Hint: What is \( X[0]? \)