Study of fast 3-D route planning approach for air vehicle

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ABSTRACT

A fast 3-D route planning method for unmanned air vehicle is proposed which can generate physically realizable 3-D route within a reasonable time. Our method includes two steps: First, 2-D route planning generates a route which satisfies turning radius constraint (abbreviated as R-constrained below); second, 3-D route planning generates 3-D route in vertical profile of the 2-D route. To make 2-D route R-constrained, a method is proposed by supposing 2-D route of air vehicle is composed of a sequence of arc route segments and tangential points between neighboring arcs are searching nodes. 3-D route planning is considered as optimal control problem, and its route can be determined by applying motion equations of air vehicle. The experiments show that our method can produce feasible 3-D routes within a reasonable time, and ensure the planned 3-D routes satisfy aerodynamics constraints of air vehicle.

Keywords: TPs, route planning, dynamic programming, A* algorithm, unmanned air vehicle

1. INTRODUCTION

Air vehicle technique has undergone substantial variations after World War II, along with development and advance in computer, automation control and information technology. Flight speed becomes fast and fast, and manoeuvrability becomes much greater than before, making its control more complex. As flight mission becomes more and more arduous and complex, the execution of flight mission becomes more and more difficult so that it can not be accomplished manually. A promising solution to this problem is to replace air vehicle by unmanned air vehicle.

One important application of unmanned air vehicle (UAV) is data gathering in large area¹, including monitoring and inspecting power lines, pipe lines, and railroad tracks; counting livestock and wildlife; measuring air quality over a board area; and profiling terrain, etc., which could be executed by teams of lightweight, low-cost UAVs requiring as few as one or two persons for launch, recovery, and monitoring. UAVs can also be substituted for pilots to execute monotonous or dangerous flight missions, such as delivering postal parcels long distance, spraying insecticide over a large farmland area, and sprinkling fire-extinguishing chemical over the scene of fire, etc.

Usually, before air vehicle carries out its mission, a optimal, at least feasible route should be planned by considering a lot of constraints such as arrival time, fuel consumption, safety and flight environment (including air traffic status, populated area, terrain, weather condition, etc.) so that air vehicle can not only complete flight mission but also return safely. At the same time, it must not endanger the order of air traffic and the safety of pedestrians. However, route planning for air vehicle has been proved to be a NP-problem, an exhaustive searching will cause combinatorial explosion.

The process of route planning can be divided into two stages: preprocess and route searching. In preprocess, information

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of flight environment is expressed mathematically by a model or planning space. Route searching stage tries to find an optimal route in planning space according to some criteria of flight mission. In order to increase the planning efficiency, a series of planning space models and planning methods were proposed in recent years. The main search space models include grid model, quadtree model and polygon model, etc. Grid model is widely employed for its simplicity, however its huge data storage limits its usage; quadtree model decreases the data storage if there are no too many obstacles in search space; polygon model is a graphics representation of search space and can decrease data storage substantially. However, in complex searching space or 3-D searching space, the complexity of computing increases greatly. Search methods in planning space can be classified as deterministic method and stochastic method. Breadth-first, depth-first, best-first algorithm and dynamic programming algorithm belong to deterministic method, their searching behaviors are predictable and repeatable, and can yield unique optimal solution for some flight mission; Stochastic method includes potential field method, genetic algorithm, simulated annealing algorithm, and Monte Carlo-Las Vegas method, etc. whose performance is unpredictable even when the same inputs are given. Therefore the optimal solutions cannot be ensured.

Section 2 introduces the configuration of our route planning system. In section 3, a 2-D route planning method is proposed which can generate R-constrained route. According to optimal control theory, in section 4, we present a 3-D route planning method. Some experiment results are presented in section 5 and our conclusion remarks are given in section 6.

2. PLANNING SPACE AND SYSTEM CONFIGURATION

Flight missions executed by air vehicles are multifarious, and the constraints to flight routes vary with flight mission. The constraints can be classified into two classes: flight environment and air vehicle maneuverability. For the sake of safety, air vehicle can only fly under some special environment. For instance, in 2-D route planning, the area with high altitude, with busy air traffic, or with dense population are not suitable for unmanned air vehicles flight, we denote them as flight-forbidden area, versus flight-free area; whereas in 3-D route planning, undoubtedly, the height of 3-D route should be higher than that of terrain. Also in order to generate a physically realizable flight route, the maneuverability of air vehicle should be further considered. When planning 2-D route, this constraint is integrated as the minimal turning radius of air vehicle, while in 3-D route planning, an air vehicle model should be considered, a simple model at least includes its maximal vertical acceleration, but a complex one will provide equations of motion of air vehicle, including mass of air vehicle, fuel consumption, propulsive force, aerodynamic force, etc.

In our route planning system, a grid model is employed as planning space. It discretizes flight area into a grid map where each grid cell represents a regular subarea. Its data structure can be an array, each dimension of array represents a physical dimension in space, and each element of the array denotes the attribute of corresponding point in flight area, e.g., flight-forbidden area or flight-free area. However, although this representation is simple, its data storage is huge. For example, a flight area of air vehicle is usually tens of thousands of square kilometers, with a resolution of 100m*100m per grid, the size of 2-D array will reach millions. If planning 3-D route, the storage will increase hundreds or thousands times further. Since the complexity of searching problem is in proportion with the size of searching space, if we plan 3-D route directly in grid space, combinatorial explosion is triggered and the planning becomes very time-consuming.

A strategy of solving complex problem is to divide the problem into several simple subproblems. Thus 3-D route planning problem can be divided into two subproblems: First, find a 2-D route which satisfies the constraints of flight environment and 2-D constraints of air vehicle. Second, search the optimal physically realizable 3-D route in the vertical profile of 2-D route. According to this strategy, a system configuration of planning is shown in Figure 1.
3. 2-D ROUTE PLANNING

Traditional research in route planning concentrated on the planning of collision-free routes for robots with high manoeuvrability. Because air vehicle requires a high flying speed to maintain its lift force, its 2-D manoeuvrability can be represented by minimal turning radius $R$ which is dependent on flight speed. We denote a route satisfying minimal turning radius as a $R$-constrained route. To obtain a $R$-constrained route for air vehicle is the first step toward planning a physically realizable 3-D route.

The problem of planning $R$-constrained route was first proposed by Dubins from the view of mathematics in the 60's, in which the set of possible solution is reduced to a discrete set of canonical trajectories. However obstacles were not considered in his approach. Based on Dubins' work, the planning problem with obstacles is analyzed in [4] where obstacles are represented by polygon model. In our planning method, a grid model is used.

3.1 Tangential Point

It is well-known that there always exists a curvature circle at every point on a continuous curve, its radius $R = C^{-1}$, where $C$ is the curvature of the curve, as shown in Figure 2. Approximately, we can assume that the curve is composed of infinitesimal arcs of those curvature circles and each of them is tangential to its neighboring arcs. On the other hand, each point on the curve is tangential point of those infinitesimal arcs. Further, if we suppose the 2-D flight state of air vehicle consists of turning and straight flying, a 2-D route of air vehicle can be regarded as a composition of arcs (straight line can be regarded as arc with infinite radius), as shown in Figure 3. Noticing that air vehicle always passes tangential points (TPs) between its passing every two arc route segments, so we take TPs as planning nodes. TPs are generated evenly dispersed on the planning space so that the minimal turning radius constraint is attached easily when we calculate arc route segments between two TPs.
The density of TPs generated in planning space affects the degree of optimization of the solution and the planning speed greatly. If all the flight-free grids of the planning space are TPs, the route obtained is the most optimal solution but the planning process is time-consuming.

3.2 Searching algorithm

In our system, 2-D route is planned by A* algorithm. The algorithm is described below:

i. Initialize OPEN={START}, CLOSE=nil, g(START)=0, f(START)=h(START), where START is the start point of A search, g(n), h(n) and f(n) are the current, predicted and total cost of node n respectively.

ii. If OPEN=nil, exit with failure. Otherwise, let current node n be the first element of OPEN (every element in OPEN list is sorted according to the value of g), remove n from OPEN, and put it in CLOSE.

iii. If n=GOAL, exit successfully with the route which can be obtained by tracing back along the pointers from n to START (pointers are assigned in step iv).

IV. Expand current node n, and evaluate every expanding subnode n'. If n' \notin OPEN or n' \in OPEN and g(n)+c(n, n') \leq g(n'), then set g(n')=g(n)+c(n, n'), assign a pointer from n' to n, and put n' into OPEN. Go to step ii.

In A* algorithm, the interface with specific application mainly includes the expansion of current node n and the evaluation of expanding subnode n' in step iv, we will discuss them in detail below.

3.3 Expansion of current node

A current node n in A* algorithm is the best searching node candidate in OPEN list, and the route from START to this node is the most optimal route. Now the problem is how to obtain the expanding subnodes of n from the TPs in planning space. A potential TPs should satisfy following rules:

1) there exists arc connects with the current node n.
2) the arc is tangential at position of node n to the optimal route from START to n.
3) the radius of the arc is larger than the minimal turning radius R.

An essential requirement of expanding method is to discard invalid TPs as many as possible but ensure the optimal solution.

Suppose turning angle of every arc route segment is less than 180 degree, according to the position of current node n and the direction of the route from START to n at n, a expanding area is defined in Figure 4. All TPs in the shaded area are selected as expanding nodes of current node n.
In Figure 4, $N_i$ denotes the current node $n$, $N_{i+1}$ is the last TP of the route from START to $n$, $O_1$, $O_2$ are the minimal turning circle, $r_{\text{min}}$ is their minimal turning radius, and $D$ is the maximal radius of expanding area which is determined according to specific application.

### 3.4 Evaluation of expanding nodes

The evaluation includes generation of route arcs from expanding node to current node $n$ and its cost calculation.

1) Generation of route arc segments

A best arc segment from expanding node to current node should be as straight as possible, in other words, its radius should be as large as possible.

As shown in Figure 5, we discretize the polar space at expanding node $N_{i+1}$ first. To each discretized angle $\theta_k$, we do the following processing:

i) Draw the tangential line of the route from START to node $n$ (e.g., $N_i$) and the straight line at $N_{i+1}$ with tilt angle $\theta_k$ (see Fig. 5). They intersect each other at $O$, denoted as $N_iO$ and $N_{i+1}O$ respectively.

ii) Calculate circle $O_{i+1}$ tangential to straight line $N_{i+1}O$ at $N_i$, which is also tangential to straight line $N_iO$.

iii) Calculate circle $O_i$ tangential to line $N_iO$ at $N_i$, which is also tangential to straight line $N_{i+1}O$.

iv) Compare the arcs of tangential circle $O_i$ and $O_{i+1}$ (e.g., $N_iP$, $N_{i+1}T$), arc with smaller radius is selected as the route (in Figure 5, the route segment is composed of line segment $N_iT$ and arc $TN_{i+1}$).

Now, we obtain all the possible arc route segments between $N_i$ and $N_{i+1}$, and turning radius constraint can be applied by comparing it with the radius of the selected arcs.

2) Evaluation of cost

In this paper, the optimization objective of 2-D route planning is to find the shortest route from start point to goal. So
the cost of the route is calculated based on the length of the route. If we expect the produced route as straight as possible, the cost calculation should take into account the frequency of turnings, radius and turning degree of route. Thus, the total cost of the route can be denoted as:

\[ C = L + c \sum \psi_i / R_i \]  

(1)

Where \( C \) is the total cost, \( L \) is the length of the route, \( \psi_i \) is the turning angle of each arc route segment, \( R_i \) is the turning radius of each arc route segment.

4. 3-D ROUTE PLANNING

3-D route planning for unmanned air vehicle can be considered as an optimal control problem: according to characteristics of aerodynamics and control of air vehicle, choose control sequence, which will make air vehicle fly in a 3-D route which can optimize the performance properties of system (such as fuel consumption, time, safety, etc.).

4.1 Optimal Control

Suppose the state equation of control system to be:

\[ \dot{x}(t) = f(x(t), u(t), t) \]  

(2)

and its initial condition is:

\[ x(t_0) = x_0 \]  

(3)

where \( x(t) \) is a n-dimensional vector, satisfying

\[ L(x(t), x(t), t) \leq 0 \]  

(4)

\( u(t) \) is a m-dimensional control vector, satisfying

\[ u(t) \in \Omega, t \in [t_0, t_f] \]  

(5)

where \( \Omega \) is a close subset of m-dimensional control function space. Choose \( \tilde{u}(t) \) satisfying (5), so that, according to system equation (2), the system state can be controlled to shift from initial state \( x_0 \) to a final state \( x_f \), optimizing system performance

\[ J = \Theta[x(t_f), t_f] + \int_{t_0}^{t_f} \Phi[x(t), u(t), t] dt \]  

(6)

4.2 Model of unmanned air vehicle

The unmanned air vehicle is supposed to be a rigid, symmetrical object. Its motion equations can be summarized as following:

\[ m \dot{v} + m \Omega \omega = F(v, \theta, u) + G(\theta) \]  

(7)

\[ I \dot{\omega} + \Omega I \omega = M_{\theta}(v, \theta, u) \]  

(8)

\[ \dot{x} = Rv \]  

(9)

\[ \dot{\theta} = E \omega \]  

(10)

where \( v = [U \ V \ W]^T \) denotes the velocity vector, \( \omega = [P \ Q \ R]^T \) is the vector of rotary angular velocity of the
moving axes. \( \mathbf{X} = [x \ y \ z]^T \) is the position vector of air vehicle, \( \mathbf{\Theta} = [\Phi \ \Theta \ \Psi]^T \) is the rotary Euler angles of the moving axes and \( \mathbf{U} \) is a \( m \times 1 \) control vector.

Let \( \mathbf{v} = \begin{bmatrix} v \\ \omega \end{bmatrix}, \mathbf{\bar{X}} = \begin{bmatrix} \bar{x} \\ \bar{\Theta} \end{bmatrix} \), accordingly

\[
\begin{align*}
M \ddot{\mathbf{v}} + N(\mathbf{v})\mathbf{v} &= C(\mathbf{v}, \mathbf{\bar{X}}, \mathbf{U}) + G(\mathbf{\bar{X}}) \quad (11) \\
\dot{\mathbf{\bar{X}}} &= \Gamma(\mathbf{\bar{X}})\mathbf{v} \quad (12)
\end{align*}
\]

where \( M = \begin{bmatrix} m1 & 0 \\ 0 & I \end{bmatrix} \) is a \( 6 \times 6 \) inertia matrix, \( N(\mathbf{v}) = \begin{bmatrix} m\Omega & 0 \\ 0 & \Omega I \end{bmatrix} \) is a \( 6 \times 6 \) matrix in relation to the rotation of moving axes, \( G(\mathbf{\bar{X}}) = \begin{bmatrix} G \\ 0 \end{bmatrix} \) is a 6 dimensional vector containing gravitational force and its moment (which here is 0),

\[
C(\mathbf{v}, \mathbf{\bar{X}}, \mathbf{u}) = \begin{bmatrix} F(\mathbf{v}, \mathbf{\Theta}, \mathbf{u}) \\ M_F(\mathbf{v}, \mathbf{\Theta}, \mathbf{u}) \end{bmatrix}
\]
contains the aerodynamic and propulsive forces and the moments they caused, and

\[
\Gamma(\mathbf{\bar{X}}) = \begin{bmatrix} R & 0 \\ 0 & E \end{bmatrix}
\]
is a \( 6 \times 6 \) transformation matrix.

Further, if we define \( \mathbf{\chi} = \begin{bmatrix} x \\ \bar{X} \\ \mathbf{v} \end{bmatrix} \) as state vector, the state space expression of air vehicle model is

\[
\dot{\mathbf{\chi}} = f(\mathbf{\chi}) + D(\mathbf{\chi}, \mathbf{u}) \quad (13)
\]

where

\[
\begin{align*}
f(\mathbf{\chi}) &= \begin{bmatrix} \Gamma(\mathbf{\bar{X}})\mathbf{v} \\ M^{-1}[-N(\mathbf{v})\mathbf{v} + G(\mathbf{\bar{X}})] \end{bmatrix} \quad (14) \\
D(\mathbf{\chi}, \mathbf{u}) &= \begin{bmatrix} 0 \\ M^{-1}C(\mathbf{v}, \mathbf{\bar{X}}, \mathbf{u}) \end{bmatrix} \quad (15)
\end{align*}
\]

In (13), control vector \( \mathbf{U} \) contains control inputs specified through the action of a human or automatic pilot which can be displacements of rudder, elevator, ailerons, spoilers, etc., when rate of climb or descent is expected to be controlled, the thrust or throttle setting should also be added.

4.3 The optimal control formulation of 3-D route planning

We formulate 3-D route planning as following optimal control description.

In a 3-D planning space, according to state equation (13) of air vehicle and initial conditions:

\[
\begin{align*}
\mathbf{\chi}(t_0) & = \mathbf{\chi}_0 \quad (16) \\
\mathbf{\chi}(t_f) & = \mathbf{\chi}_f \quad (17)
\end{align*}
\]

where \( \mathbf{\chi}_0, \mathbf{\chi}_f \) are start point and goal point respectively, find a set of control sequences which can make air vehicle fly from the start point to the goal point satisfying constraints \( \tilde{L}(\mathbf{\chi}, \dot{\mathbf{\chi}}) = 0 \) (such as, height of 3-D route must be higher than that
of terrain, etc.), and its performance description $J$ is optimized.

If the fuel consumption is expected optimal, the performance description is

$$J = \int m \, dt.$$  \hspace{1cm} (18)

where $m$ is the rate of fuel consumption.

If the arrival time of air vehicle is optimal, let

$$J = \int 1 \, dt.$$  \hspace{1cm} (19)

And if we needs a lowest observability route, let

$$J = \int S(x, \dot{x}) \, dt$$ \hspace{1cm} (20)

where $S(x, \dot{x})$ is a observability function.

### 4.4 Realization of 3-D route planning

In computer, the problem of optimal control is solved by employing numerical methods (for example, conjugate gradient penalty method, the Newton-Raphson method, control vector parameterization method 5,7, or dynamic programming method 8). In our 3-D planning system, we prefer dynamic programming which is more advanced. As we have stated before, the 3-D planning proceeds on the vertical profile of obtained 2-D R-constrained route as shown in Figure 5. We sample the height along the 2-D route from start point to goal point producing a multi-stage decision space, in which dynamic programming method is applied.

![3-D route planning](image)

**Figure 5. 3-D route planning**

### 5. EXPERIMENT RESULT

In our experiment, the planning area is a 256*256 digital terrain map produced by simulation. Our planning objective are to generate a shortest 2-D route and make air vehicle flight as low as possible but must satisfy the safety. In 3-D route planning, we simplify the air vehicle model by supposing control system of air vehicle to work ideally and rotation of air vehicle around its centroid to be ignored in flight. We also suppose the control vector contains only elevator angle and rudder angle. The experiment is made on a IBM PC 80586 (100Hz) and the planning time is 56 seconds for 2-D route, and 32 minutes for 3-D route. Some results are shown below.

In Figure 6, black area is flight-free area whereas gray and white area are flight-forbidden areas with different attributes. White points evenly dispersed in flight-free area are TPs, and black dots marked on the white 2-D route is the selected tangential points between neighboring arc route segments.
The vertical profile of 2-D route and 3-D route obtained in it are shown in Figure 7, where the vertical line area indicates terrain altitude of the area where 2-D route passes. The sample span on the vertical profile of 2-D route is 10, and the whole route is sampled into 22 stages. The gray area shows the 3-D route candidates, and the final 3-D route is the lowest black curve above terrain.

Figure 6. 2-D route planning result

Figure 7. Vertical profile of 2-D route and generated 3-D route

Figure 8. Elevator angle and rudder angle
The control sequences of elevator and rudder are shown in Figure 8, and a projected view of the 3-D route is shown in Figure 9.

6. CONCLUSION

In this paper, a fast 3-D route planning method for unmanned air vehicle is presented, which divides 3-D route planning into two steps, a 2-D route planning and a 3-D route planning in profile of 2-D route. In 2-D route planning, we present a method which can generate 2-D R-constrained route in grid space. Then, we analyzes the 3-D route planning problem from the view of optimal control. Based on this analysis, 3-D route planning problem can be solved by using dynamic programming approach.

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