Exercise 1, Page 134:
In the TOYCO model, suppose that the changes $D_1$, $D_2$, and $D_3$ are made simultaneously in the three operations.

(a) If the availabilities of operations 1, 2, and 3 are changed to 438, 500, and 410 minutes, respectively, use the simultaneous conditions to show that the current basic solution remains feasible, and determine the change in the optimal revenue by using the optimal dual prices.

(b) If the availabilities of operations are changed to 460, 440, and 380 minutes, respectively, use the simultaneous conditions to show that the current basic solution becomes infeasible.

Exercise 3, page 135:
A company produces three products, $A$, $B$, and $C$. The sales volume for $A$ is at least 50% of the total sales of all three product. However, the company cannot sell more than 75 units of $A$ per day. The three products use one raw material, of which the maximum daily availability is 240lb. The usage rates of the raw material are 2 lb per unit of $A$, 4 lb per unit of $B$, and 3 lb per unit of $C$. The unit prices for $A$, $B$, $C$ are $20$, $50$, and $35$, respectively.

(a) Determine the optimal product mix for the company.

(b) Determine the dual price of the raw material resource and its allowable range. If available raw material is increased by 120 lb, determine the optimal solution and the change in total revenue using the dual price.

(c) Use the dual price to determine the effect of changing the maximum demand for product $A$ by $\pm 10$ units.

Exercise 1, page 144:
In the TOYCO model, determine if the current solution will change in each of the following cases:
(a) \( z = 2x_1 + x_2 + 4x_3 \).
(b) \( z = 3x_1 + 6x_2 + x_3 \).
(c) \( z = 8x_1 + 3x_2 + 9x_3 \).

**Exercise I:** Suppose you are given the dependencies of a solution on the changes in two resources as follows:

\[
x_1 = 2 + \frac{2}{3}D_1 + \frac{1}{3}D_2,
\]

\[
x_2 = 2 - \frac{1}{3}D_1 + \frac{2}{3}D_2 \geq 0.
\]

(a) Determine the optimal solution.
(b) For what ranges of \( D_1 \) and \( D_2 \), the given basis remains feasible?
(c) Suppose that the two resources are increased simultaneously by \( \delta \). What is the range for \( \delta \) for which the basis remains optimal?

**Exercise II:** A production plant is making products \( x_1, x_2, x_3 \) with profits $24, $22, and $45, respectively. Each product has to be run through 3 machines with daily limits on the operation hours. The current problem of optimizing profit is given by

\[
\begin{align*}
\text{maximize} & \quad z = 24x_1 + 22x_2 + 45x_3 \\
\text{subject to} & \quad 2x_1 + x_2 + 3x_3 \leq 42 \quad \text{(machine 1)} \\
& \quad 2x_1 + x_2 + 2x_3 \leq 40 \quad \text{(machine 2)} \\
& \quad x_1 + \frac{1}{2}x_2 + x_3 \leq 45 \quad \text{(machine 3)} \\
& \quad x_1, x_2, x_3 \geq 0
\end{align*}
\]

The current optimal production plan is summarized in the following table:

<table>
<thead>
<tr>
<th>Basic</th>
<th>( x_1 )</th>
<th>( x_2 )</th>
<th>( x_3 )</th>
<th>( x_4 )</th>
<th>( x_5 )</th>
<th>( x_6 )</th>
<th>RHS</th>
</tr>
</thead>
<tbody>
<tr>
<td>( z )</td>
<td>20</td>
<td>0</td>
<td>0</td>
<td>1</td>
<td>21</td>
<td>0</td>
<td>882</td>
</tr>
<tr>
<td>( x_3 )</td>
<td>0</td>
<td>0</td>
<td>1</td>
<td>1</td>
<td>-1</td>
<td>0</td>
<td>2</td>
</tr>
<tr>
<td>( x_2 )</td>
<td>2</td>
<td>1</td>
<td>0</td>
<td>-2</td>
<td>3</td>
<td>0</td>
<td>36</td>
</tr>
<tr>
<td>( x_6 )</td>
<td>0</td>
<td>0</td>
<td>0</td>
<td>0</td>
<td>-\frac{1}{2}</td>
<td>1</td>
<td>25</td>
</tr>
</tbody>
</table>
where $x_4, x_5$ and $x_6$ are slack variables for machines 1, 2, and 3, respectively.

The company wants to improve the currently optimal profit, without changing current optimal basis. It considers two possible strategies:

(1) Extend the operation hours on one of the machines so as to improve the profit currently made (current optimal). However, increasing the operation hours comes at the cost of $3 per operating hour for any of the machines and there is a budget of $9 dollars for the investment.

(2) Increase the profit (price) on one of the products.

Which of the two plans gives the company the most increase in the profit?