Homework 2

Due: September 11, 2009

Exercise 4, page 83:
Two different products, P1 and P2, can be manufactured by any of the two different machines, M1 and M2. The unit processing time of either product on either machine is the same. The total daily capacity of machine M1 is 200 units, while the total daily capacity of machine M2 is 250 units. The shop supervisor wants to balance the production schedule on the two machines such that the total number of units produced on one machine is within 5 units of the number produced on the other. The profit per unit of P1 is $10 and that of P2 is $15. Set up the problem as an LP in standard form.

Modified Exercise 5, page 83:
Consider the following problem

\[
\begin{align*}
\text{minimize} & \quad z = \max\{|x_1 - x_2 + 3x_3|, | - x_1 + 3x_2 - x_3|\} \\
\text{subject to} & \quad x_1, x_2, x_3 \geq 0.
\end{align*}
\]

Formulate the problem as an LP in standard form.

Exercise 1, page 89:
Consider the following LP

\[
\begin{align*}
\text{maximize} & \quad z = 2x_1 + 3x_2 \\
\text{subject to} & \quad x_1 + 3x_2 \leq 6 \\
& \quad 3x_1 + 2x_2 \leq 6 \\
& \quad x_1, x_2 \geq 0.
\end{align*}
\]

(a) Express the problem as a standard LP.

(b) Determine all the basic solutions of the problem and classify them as feasible or infeasible.

(c) Use direct substitution in the objective function to determine the optimal basic feasible solution.

(d) Verify graphically that the solution in (c) is indeed optimal.

(e) Show how the infeasible basic solutions are represented on the graphical solution space.
Modified Exercise 6, page 101:
The following tableau represents a specific simplex iteration. All variables are non-negative. The tableau is not optimal for either maximization or a minimization problem.

<table>
<thead>
<tr>
<th>Basic</th>
<th>$x_1$</th>
<th>$x_2$</th>
<th>$x_3$</th>
<th>$x_4$</th>
<th>$x_5$</th>
<th>$x_6$</th>
<th>$x_7$</th>
<th>$x_8$</th>
<th>Solution</th>
</tr>
</thead>
<tbody>
<tr>
<td>$z$</td>
<td>0</td>
<td>-5</td>
<td>0</td>
<td>4</td>
<td>-1</td>
<td>-10</td>
<td>0</td>
<td>0</td>
<td>620</td>
</tr>
<tr>
<td>$x_8$</td>
<td>0</td>
<td>3</td>
<td>0</td>
<td>-2</td>
<td>-3</td>
<td>-1</td>
<td>5</td>
<td>1</td>
<td>12</td>
</tr>
<tr>
<td>$x_3$</td>
<td>0</td>
<td>1</td>
<td>1</td>
<td>3</td>
<td>1</td>
<td>0</td>
<td>3</td>
<td>0</td>
<td>6</td>
</tr>
<tr>
<td>$x_1$</td>
<td>1</td>
<td>-1</td>
<td>0</td>
<td>0</td>
<td>6</td>
<td>-4</td>
<td>0</td>
<td>0</td>
<td>0</td>
</tr>
</tbody>
</table>

(a) Categorize the variables as basic and nonbasic, and provide the current values of all the variables.

(b) Assuming that the problem is of the maximization type, identify the nonbasic variables that have the potential to improve the value of $z$. If each such variable enters the basic solution, determine the associated leaving variable (if any) and the associated change in $z$.

(c) Repeat part (b) assuming that the problem is of the minimization type.

(d) Which nonbasic variable(s) will not cause a change in the value of $z$ when selected to enter the solution.

Exercise 3, page 111
Solve Problem 5 (a) of Set 3.4A (page 107) by the two-phase method.

Exercise 1, page 118
For the following LP, identify three alternative optimal basic solutions, and then write a general expression for all the nonbasic alternative optima comprising these three basis solutions:

maximize $z = x_1 + 2x_2 + 3x_3$
subject to $x_1 + 2x_2 + 3x_3 \leq 10$
\hspace{1cm} $x_1 + x_2 \leq 5$
\hspace{1cm} $x_1 \leq 1$
\hspace{1cm} $x_1, x_2, x_3 \geq 0$.

Note: Although the problem has more than three alternative basic solution optima, you are only required to identify three of them.