NOTE: Please show each iteration of your algorithm. No credit will be given for writing the final answer only.

**Problem 1**
Consider the network in Figure 1.

(a) Formulate the shortest path problem from node $s$ to node $t$ as an ILP problem.

(b) Consider the matrix associated with the equality constraints in your formulation. Is the matrix totally unimodular?

(c) What can you conclude about the problem solution?

**Problem 2**
Using ILP model, formulate the following requirements:

(a) At least one of the following two inequalities holds:

$$3x_1 - x_2 - x_2 + x_4 \leq 12$$
$$x_1 + x_2 + x_3 + x_4 \leq 15$$
(b) **Either** $5x_1 + 3x_2 + 6x_3 \leq 100$ **or** $4x_1 + 6x_2 + 3x_3 \leq 100$

(c) $3x_1 + 2x_2 = 6$ **or** $3x_1 + 2x_2 = 12$ **or** $3x_1 + 2x_2 = 18$

**Problem 3**
Suppose we have 4 projects to choose from. Using ILP model, formulate the following requirements:

(a) No more than two projects can be selected

(b) Either project 1 or 2 can be selected only if either 3 or 4 are selected

(c) Project 1 has to be selected when projects 2, 3 and 4 are selected

**Problem 4**
Solve the following problem by Branch-and-Bound algorithm.

\[
\begin{align*}
\text{maximize} & \quad 18x_1 + 14x_2 + 8x_3 + 4x_4 \\
\text{subject to} & \quad 15x_1 + 12x_2 + 7x_3 + 4x_4 + x_5 \leq 37 \\
& \quad x_1, x_2, x_3, x_4, x_5 \in \{0, 1\}
\end{align*}
\]