IE 598 AS: Game-Theory: Models, Algorithms and Applications

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MW 1.00-2.40, Room 110, Speech and Hearing Clinic

This course provides an introduction to game-theoretic models, with a focus on the theory and algorithms for the solution of equilibrium problems over *continuous strategy sets*. Specifically, we will develop convergence theory for centralized and distributed approaches for the solution of a variety of game-theoretic problems. Course topics will include fixed-point theorems, Nash equilibrium problems, generalized Nash equilibrium problems and Stackelberg equilibrium problems. We will draw on applications from communication networks, electricity markets, supply-chain networks and traffic equilibrium problems. Students will be expected to implement some of the algorithms on Matlab.

Apart from students in IESE, this course would be of interest to students from math, ECE, civil engineering, chemical engineering, computer science, general engineering, economics and the operations management program in the business school. No prior background in optimization or game-theory is necessary. However, students would be required to have some background in multivariate calculus and linear algebra. The following books will be adhered to with volume 1 of the first set of references serving as the class text.

- (FP) Facchinei, F. and Pang, J-S, *Finite-dimensional variational inequalities and complementarity problems*, Vols. I and II. Springer Series in Operations Research. Springer-Verlag, New York, 2003
- (CPS) Cottle, R. W. and Pang, J-S and Stone. R. E., *The Linear Complementarity Problem*, Computer Science and Scientific Computing. Academic Press, Inc., Boston, MA, 1992.
- (LPR) Luo, Z-Q, Pang, J-S and Ralph, D., *Mathematical programs with Equilibrium Constraints*, Cambridge University Press, Cambridge, 1996.
- (MWG) Mas-Colell, A., Whinston, M.D. and Green, J.R. *Microeconomic Theory* , Oxford University Press, USA, 1995.
- (B) Bertsekas D. P., *Nonlinear Programming*, Athena Scientific, 1995.
- (BON) Bertsekas Dimitri P., Ozdaglar, A. and Nedich, A., *Convex Optimization and Analysis*, Athena Scientific, 2003.
- (KON) Konnov, I.V. *Equilibrium Models and Variational Inequalities*, Elsevier, 2007.

A tentative course outline is provided below with specifications of the main references and the approximate number of lecture hours to cover a specific topic.

1. (B,BON) Background: Optimization theory and algorithms (convexity, first and second-order KKT conditions, regularity conditions, duality theory, quick survey of algorithms). (2)

2. Nash equilibrium models
   (a) (MWG, KON) Nash equilibrium: Background (1)
   (b) (FP-I,CPS) Applications and models: Linear complementarity problems (LCPs), nonlinear complementarity problems (NCPs) and variational inequalities (VIs) (2)
   (c) (FP-I,CPS) Existence and uniqueness of equilibria (3)
   (d) Algorithms:
      i. (CPS) Lemke's method (pivot method for LCPs) (1)
      ii. (FP-II) Interior-point approaches (2)
      iii. (FP-II) Sequential linearization approaches (1)
3. Generalized Nash equilibrium models
   (a) Applications and models: Quasi-variational inequalities (QVIs) and mixed complementarity problems (mCPs) (2)
   (b) (CPS,FP-I) Existence and uniqueness of equilibria (2)
   (c) Algorithms
      i. (FP-II) Penalization approaches (2)
4. Stackelberg equilibrium models
   (a) (LPR) Applications and models: Mathematical programs with complementarity constraints (MPCCs) (2)
   (b) (LPR) Strong stationarity and second-order conditions (2)
   (c) Algorithms
      i. (LPR) Sequential quadratic programming approaches (2)
      ii. (LPR) Interior-smoothing and Interior-penalty approaches (2)

The course grade will be based on homeworks (30%), an examination (40%) and a project (30%). Often the homeworks may include some programming assignments. These may be coded in Matlab or any other programming language. The project will involve studying a set of papers pertaining to a particular topic and presenting them in an oral and written form.