Multiuser Detection in DS-CDMA

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Abstract

The theory of Multiuser Detection (MUD) for DS-CDMA is studied. A brief description of optimal MUD is given and some low complexity sub optimal implementations are described. This is followed by a short note on Blind Multiuser detection. Conclusions are drawn from theoretical and simulation results. Towards the end, a new low complexity scheme for MUD in DS-CMDA is proposed. Simulation results show that the performance of the proposed algorithm compares with the optimal maximum-likelihood (ML) MUD under various situations.

Index Terms

Multiuser Detection (MUD), multiple access interference (MAI), maximum-likelihood (ML) detection, log likelihood ratio (LLR)

I. INTRODUCTION

The performance of a DS-CDMA system is limited by Multiple Access Interference (MAI) and the Near Far Problem. MAI gives rise to irreducible error even in absence of thermal noise, while near far problem arises since high power users destroy the communication of low power users. Use of orthogonal codes is not considered to be a good solution to this problem since multipath fading and delay destroys the orthogonality of the signature waveforms. Moreover, the theoretical limit on the number of orthogonal codes for a fixed spreading gain restricts the number of users in the system.

The conventional decoder treats the signals of all the other users as noise and tries to suppress it. Thus, conventional decoding requires that the interference from other users should be minimal. This places the entire burden of performance on the cross correlation property of the spreading codes of the users. However, the interference suppression capability of such system deteriorates as the number of users grows in the system.

A better detection strategy is to jointly detect multiple users, where the additional structure of the MAI is exploited rather than considered as noise. Multiuser Detection deals with the demodulation of the digitally modulated signals in the presence of MAI.

The optimal maximum likelihood (ML) receiver for multi-user detection (MUD) was found by Verdu in [1]. It showed an improvement over conventional decoder by orders of magnitude. However, the practical implementation of such scheme is limited by the decoding complexity which grows exponentially with the number of users.

A class of linear receives and suboptimal receiver are described in [2]-[3] as a trade off between complexity and performance. The main idea is to use some appropriate linear transformations on the outputs of a matched filter bank.

The above scheme makes too many assumptions about what is known at receivers (signature and timing information of desired user and interferer, received amplitudes etc). This is can be implemented in uplink channels but it is not practical in downlink channels. The downlink receiver is generally limited in terms of power, complexity and memory and since a downlink receiver needs to detect the bits of only a particular user, joint detection is not energy efficient. It was shown in [4] and [5] that *blind* implementation of some linear MUD schemes is possible which require knowledge no more than that required by a conventional detector (only desired user's waveform and its timing).

With this motivation, we study the theory of Multiuser detection (MUD), existing algorithms and current implementation issues. The papers that we study are the key foundations of Multiuser Detection. We also suggest a low complexity MUD scheme and compare its performance with the existing schemes.

This report is organized as follows. In section II, the CDMA system model is presented. A literature survey on MUD is presented in section III which forms the major part of this report. A new low complexity MUD is proposed in section IV. Simulation results are presented in Section V and conclusions are given in section VI.

II. System Model

Consider a CDMA system shared by K asynchronous users simultaneously. We consider BPSK transmission through a common AWGN channel. Each user is assigned a unique signature waveform $s_k(t)$ of duration T, where T is the symbol duration. A signature waveform can be expressed as

$$s_k(t) = \sum_{j=0}^{N-1} a_k(n) p_{T_c}(t - nT_c), 0 \leq t \leq T \tag{1}$$

where $\{a_k(n) \in \{-1, +1\}, 0 \leq n \leq N-1\}$ is a spreading sequence of the k^{th} user, $p_{T_c}(t)$ is a pulse of duration T_c, where T_c is the chip duration. Thus $N = T/T_c$ is the processing gain of each users. We normalize the signature waveform so that their autocorrelation is 1 and we assume use of normalized signature waveform without change of notation in further treatment. The baseband received signal $r(t)$ can be written as

$$r(t) = \sum_{k=1}^{K} A_k \sum_{i=1}^{M} b_k(i) s_k(t - iT - \tau_k) + n(t) \tag{2}$$

where A_k denotes the received amplitude, M is the number of bits transmitted by each user, $n(t)$ is gaussian noise with power spectral density $N_o/2$, $b_k(i) \in \{-1, +1\}$ is the i^{th} transmitted bit of the k^{th} user and τ_k is he delay of the k^{th} user. Without loss of generality, the users are assumed to be ordered such that $0 \leq \tau_1 \leq \tau_2 \leq \ldots \tau_k \leq T$.

The problem is to observe $r(t)$ and to detect the transmitted bits such that the error probability is minimum. The first step is to reduce $r(t)$ to a set of vector forming a sufficient statistics for $\mathbf{b}$. A sufficient statistic of $r(t)$ is the sampled output of the matched filter (MF) of all the users for the whole interval. The MF output for the k^{th} user during the i^{th} signal interval is

$$y_k(i) = \int_{iT+\tau_k}^{(i+1)T+\tau_k} r(t) s_k(t - iT - \tau_k) dt \tag{3}$$

Using vector notation, the LK matched filter output can be considered as

$$\mathbf{y} = \mathbf{RAb} + \mathbf{n} \tag{4}$$

where

$$\mathbf{y} = [\mathbf{y(1)^T}, \mathbf{y(2)^T}, \ldots, \mathbf{y(M)^T}]^\mathbf{T} \in \mathbf{R^{MK}}, \qquad \mathbf{y(i)} = [\mathbf{y_1(i)}, \mathbf{y_2(i)}, \ldots, \mathbf{y_k(i)}]^\mathbf{T} \in \mathbf{R^K},$$

$$\mathbf{b} = [\mathbf{b(1)^T}, \mathbf{b(2)^T}, \ldots, \mathbf{b(M)^T}]^\mathbf{T} \in \{\mathbf{-1}, \mathbf{+1}\}^\mathbf{MK}, \quad \mathbf{b(i)} = [\mathbf{b_1(i)}, \mathbf{b_2(i)}, \ldots, \mathbf{b_k(i)}]^\mathbf{T} \in \{\mathbf{-1}, \mathbf{+1}\}^\mathbf{K},$$

$$\mathbf{n} = [\mathbf{n(1)^T}, \mathbf{n(2)^T}, \ldots, \mathbf{n(M)^T}]^\mathbf{T} \in \mathbf{R^{MK}}, \qquad \mathbf{n(i)} = [\mathbf{n_1(i)}, \mathbf{n_2(i)}, \ldots, \mathbf{n_k(i)}]^\mathbf{T} \in \mathbf{R^K},$$

$\mathbf{A}$ is matrix that contains the received amplitudes of the users, and $\mathbf{R}$ is a $KL \times KL$ correlation matrix

$$\mathbf{R} = \begin{bmatrix} \mathbf{R(0)} & \mathbf{R(1)^T} & \mathbf{0} & \ldots & \ldots & \mathbf{0} \\ \mathbf{R(1)} & \mathbf{R(0)} & \mathbf{R(1)^T} & \mathbf{0} & \ldots & \mathbf{0} \\ \vdots & \vdots & \vdots & \vdots & \vdots & \vdots \\ \mathbf{0} & \mathbf{0} & \mathbf{0} & \mathbf{R(1)} & \mathbf{R(0)} & \mathbf{R(1)^T} \\ \mathbf{0} & \mathbf{0} & \mathbf{0} & \mathbf{0} & \mathbf{R(1)} & \mathbf{R(0)} \end{bmatrix}$$

where $\mathbf{R(m)}$ is a $K \times K$ matrix with elements

$$R_{k,l} = \int_{-\infty}^{+\infty} s_k(t - \tau_k) s(t + mT - \tau_l) dt \tag{5}$$

The Gaussian noise vector $\mathbf{n}$ has zero mean and autocorrelation matrix $E[\mathbf{n}\mathbf{n}^\mathbf{T}] = \frac{\mathbf{N_o}}{\mathbf{2}}\mathbf{R}$. This model can also be extended to include multipath effect.

For synchronous DS-CDMA, $\tau_1 = \tau_2 = \ldots = \tau_K = 0$. Since the symbol boundaries of each of the user s is aligned,it is sufficient to consider just one symbol interval and received signal model becomes

$$r(t) = \sum_{k=1}^{K} A_k b_k s_k(t) + n(t), t \in [0, T] \tag{6}$$

and the vector notation *simplifies* to $\mathbf{y} = \mathbf{RAb} + \mathbf{n}$ where $\mathbf{y}$ is now a $K \times 1$ vector of matched filter outputs of K users, $\mathbf{b}$ is a $K \times 1$ vector of tarnsmitted bits, $\mathbf{R}$ is the $K \times K$ correlation matrix with $(i, j)^{th}$ entry being ρ_{ij}, $\mathbf{A}$ is a $K \times K$ diagonal matrix with k^{th} diagonal entry being the received amplitude of user k and $\mathbf{n}$ is a $K \times 1$ noise vector with $E[\mathbf{n}] = \mathbf{0}$ and $E[\mathbf{n}\mathbf{n}^\mathbf{T}] = \frac{\mathbf{N_o}}{\mathbf{2}}\mathbf{R} = \sigma^\mathbf{2}\mathbf{R}$.

III. Literature Survey

In this section, we present a summary of [1]-[5] in terms of main ideas, assumptions, results obtained, approaches and comments. Through most of the treatment is for synchronous DS-CDMA, for most of the cases results can be easily extended to the asynchronous case without much difficulty.

A. Optimum Multiuser Detection

It was shown in [1] that optimum multiuser detector for asynchronous Gaussian Multiple-Access Channel is essentially a K user maximum-likelihood sequence detector which consists of a bank of single user matched filter followed by a Viterbi Algorithm whose complexity per bit is $O(2^K)$. Moreover, bound on error probability for such detector is derived. It introduced two new performance measures for multiuser receiver: *Asymptotic Efficiency* and *Near-Far Resistance*.

(A.1) *Assumptions*:

It is assumed that following is known at the receiver: (1) The signature waveform of all the users, (2) The timing (bit-epoch and carrier phase) of all the users and, (3) The received amplitudes of all the users. Moreover, an AWGN channel is considered and bits are assumed to be independent and equally likely to be $+1$ or -1.

(A.2) *Main Idea*:

The key idea was that observation of the whole waveform is required to produce a sufficient statistics for any symbol decision. Observation of the complete intervals of the overlapping symbols of the other users gives additional information about the received signal in the bit interval in question. Moreover, form a single user point of view, the known structure of interference (signature waveform and timing) can be exploited for the detection purpose. Hence a joint detection over the entire obersvation period performs much better than the conventional detector which is optimal only when nothing is known about the interfering signal. Thus, a bank of single user matched filter output $\mathbf{y}$ given by (3) forms a sufficient statistics for detection of $b_k(i)$ and *NOT* $y_k(i)$ alone.

(A.3) *Results and Approaches*:

It was shown in [1] that optimum multiurer detector selects the hypothesis $\hat{\mathbf{b}}$ which corresponds to selecting the noise realization with minimum energy. For the simple case of synchronous DS-CDMA, this reduces to

$$\hat{\mathbf{b}} = \ arg \min_{\mathbf{b}\in\{-\mathbf{1},+\mathbf{1}\}^{\mathbf{K}}} \int_0^T \left(r(t) - \sum_{k=1}^{K} A_k b_k s_k(t) \right)^2 dt \tag{7}$$

$$= arg \max_{\mathbf{b}\in\{-\mathbf{1},+\mathbf{1}\}^{\mathbf{K}}} 2\mathbf{b^T A y} - \mathbf{b^T A R A b} \tag{8}$$

This can also be obtained by performing maximum likelihood detection for $\mathbf{b}$ and using the fact that $\mathbf{b}|\mathbf{y}$ follows multivariate gaussian distribution with mean $\mathbf{RAb}$ and covariance matrix $\frac{N_o}{2}\mathbf{R}$. Moreover, an efficient maximization of this objective function for a general case of asynchronous DS-CDMA can be done by applying a Viterbi algorithm which has 2^{K-1} states and per bit complexity of 2^K. The trick is to rewrite the objective function to be maximized as

$$\Omega(\mathbf{b}) = \sum_{\mathbf{n=1}}^{\mathbf{N}} \sum_{\mathbf{k=1}}^{\mathbf{K}} \lambda_{\mathbf{n,k}}(\mathbf{b}) \tag{9}$$

where $\Omega(\mathbf{b})$ is the objective function to be maximized as given by (7), $\lambda_{n,k}$ is a metric term that depends *only* on the data symbol at or above $b_k(n)$ in $\mathbf{b}$. This allows us to apply the dynamic programming for maximizing $\Omega(\mathbf{b})$.

It is not possible to find the exact error probability since it involves integration over a multivatiate Gaussian distribution. However [1] gives the bounds on error probability and showed that they converge for high SNR. This is based on the following observation:

$$P[\mathbf{b} \rightarrow \hat{\mathbf{b}}] = \mathbf{P}[\mathbf{\Omega}(\hat{\mathbf{b}}) = \max_{\mathbf{d}} \mathbf{\Omega}(\mathbf{d})] \geq \mathbf{P}[\mathbf{\Omega}(\hat{\mathbf{b}}) \geq \mathbf{\Omega}(\mathbf{b})] \tag{10}$$

The right hand side of the quation can be computed more conveniently than the left hand side. This is used to obtain the upper bound on the error probability. This is done by taking decomposition of error vectors. Moreover, for finding a lower bound, a receiver is constructed with some side information such that the error probability for such receiver can easily be computed. The actual error probability is grater than that for the receiver with side information. For high SNR, the approximate error probabiliy for K^{th} user is given by

$$P_k^{opt} \approx \max_{\epsilon \in \{-1,0,1\}^K} : 2^{1-w(\epsilon)} Q\left(\frac{\epsilon^T \mathbf{ARA}\epsilon}{\sqrt{\sigma^2 \epsilon^T \mathbf{ARA}\epsilon}}\right) \tag{11}$$

where $w(\epsilon) = \sum_{k=1}^{K} |\epsilon_k|$ is the weight of an error vector.

Two new performance measure for Multiuser Detector were introduced in [1]. The *Asymptotic Efficiency* is defined as the ratio of SNR required to achieve the same probability of error in absence of interferers to the actual SNR (not including intefering users) in the limit of background Gaussian noise level going zero. This characterizes the performance loss due to MAI. Mathematically, the k^{th} user Asymptotic Efficiency of a detector whose k^{th} user error probability and energy are equal to P_k and A_k^2 respectively is given by

$$\eta_k = sup\left(0 \leq r \leq : \lim_{\sigma \rightarrow 0} P_k(\sigma)/Q\left(\frac{\sqrt{rA_k^2}}{\sigma}\right) < +\infty\right) \tag{12}$$

where, σ^2 is the Gaussian Noise power. Similarly, *Near-far Resistasnce* characterizes the performance imparement due to unequal power of users. This is equal to the Asymptotic Efficiency minimized over the engergies of all the interfering users.

For conventional receiver, error probability for k^{th} user P_k is given by

$$\begin{aligned}
P_k^c &= P[y_k > 0 | b_k = -1]P[b_k = -1] + P[y_k < 0 | b_k = +1]P[b_k = +1] \\
&= P[y_k < 0 | b_k = +1] \\
&= \sum_{\substack{\mathbf{b} \in \{-\mathbf{1},+\mathbf{1}\}^{\mathbf{K}} \\ b_k = +1}} P[y_k < 0 | \mathbf{b}]\mathbf{P}[\mathbf{b} | \mathbf{b_k} = +\mathbf{1}] \\
&= 2^{1-K} \sum_{\substack{\mathbf{b} \in \{-\mathbf{1},+\mathbf{1}\}^{\mathbf{K}} \\ b_k = +1}} Q\left(\frac{A_k - \sum_{i \neq k} A_i b_i \rho_{ik}}{\sigma}\right)
\end{aligned}$$

and it can be easily shown that for a conventional receiver Asymptotic Efficiency is given by

$$\eta_k^c = max^2 \left(0, 1 - \sum_{i \neq j} |\rho_{ik}| \frac{A_i}{A_k} \right) \tag{13}$$

which implies that Near-far resistance, $\bar{\eta^c}_k$ is zero if signals are non orthogonal. In contrast, for optimal receiver,

$$\eta_k^{Opt} = \frac{1}{A_k^2} \min_{\substack{\epsilon \in \{-1,0,+1\}^K \\ \epsilon_k = 1}} \epsilon^T \mathbf{ARA} \epsilon \tag{14}$$

$$\bar{\eta_k}^{Opt} = \frac{1}{R_{kk}^+} \tag{15}$$

where $\mathbf{R}^+$ is the generalized Moore Penrose inverse of $\mathbf{R}$.

Even though computation of η_k is not easy, a close form expression for $\bar{\eta_k}$ can be obatined. The trick to this is to reduce the combinatorial optimization problem for calculation of η_k to a continuous optimization problem.

Further, it is also shown in [2] that the optimization problem (7) and (14) is NP hard.

(A.4) *Comments*:

$[1]$ introduce a whole lot of new concept about Multiuser Detection. For non orthogonal signals, the conventional receiver is shown to have very poor performance in terms of asymptotic Efficiency and near far resistance while the Optimal ML-MUD scheme is shown is perform much better. However, the computational complexity and delay due to obserbing the entire reveied signal before processing prohibits its practical implementations. Moreover, comparing the performance of Multiuser receiver in terms of Asymptotic Efficiency and Near-far resistance captures the problem of MAI and unequal power. This is a much better approach since in most of the cases it is not possible to obtain the closed form expression for BER, while a closed form expression can be obtained for Asymptotic Efficiency and Near-far resistance. The ideas presented in [1] pioneered the whole new field of Multiuser Detection and inspired many attempts to find low complexity sub optimal schemes which we discuss next.

The optimal MUD makes too many assumptions about what is known to the receiver (timing, phase, received amplitudes etc). It is difficult to estimate all these quantities accurately in a real environment. A possible extension to this work can to analyze the performance of the Optimal receiver in presence of estimation error of these quantities.

B. Low complexity Multiuser detectors

Since the complexity of the optimal MUD scheme grows exponentially with number of users, low complexity receivers are required for practical implementations. A family of linear suboptimal detectors are introduced in [2],[3]. The general structure of these detectors consists of a bank of matched filters, a linear transformation on matched filter outputs followed by a thresholding unit.

$$\mathbf{Ly} = \mathbf{LRAb} + \mathbf{Ln}$$

$$\hat{\mathbf{b}} = sgn(\mathbf{Ly})$$

$$E[\mathbf{Ln}] = \mathbf{0}; \quad \mathbf{E}[(\mathbf{Ln})(\mathbf{Ln})^{\mathbf{T}}] = \sigma^{\mathbf{2}}\mathbf{LRL}^{\mathbf{T}}$$

(B.1) *Assumptions*:

The results are presented for DS-CDMA on AWGN channel. Knowledge of the signature waveform of all the users, timing (bit-epoch and carrier phase) of all the users and, received amplitude of signal of user are assumed to be known. Last assumption is *not* required for the Decorrelating detector while MMSE detector requires an *additional* information about the Gaussian noise power.

(B.2) *Main Idea*:

The results in [2],[3] are based on the idea that a set of appropriately chosen memeoryless linear transformations on a matched filter output is expected to give better performance than a conventional receiver. Moreover, [3] draws similarity between the problem of MUD and Equlizations techniques developed for mitigating ISI and tries to apply some of the techniques developed for equalization for MUD.

(B.3) *Results and Approaches*:

Two different detectors are proposed in [2],[3] based on different optimization criterion.

Decorrelating Detector

In absence of noise, the received signal is given by $\mathbf{y} = \mathbf{RAb}$, so a natural strategy would be to use the detection rule $\hat{\mathbf{b}} = \mathbf{R}^{-\mathbf{1}}\mathbf{y}$. This detector doesn't require the energy of the users for detection. [2] shows that it is a solution to the maximum likelihood detector when the received amplitudes are not known by the receiver. It proposed the following optimization problem for synchronous DS-CDMA and solved it by reducing a combinatorial optimization problem to a continuous optimization problem:

$$\begin{aligned}\hat{\mathbf{b}} &= arg \min_{\mathbf{b}\in\{\mathbf{-1},\mathbf{+1}\}^{\mathbf{K}}} \min_{\substack{A_i>0\\ i=1,\ldots,K}} \int_0^T \left(r(t) - \sum_{k=1}^{K} b_k s_k(t)\right)^2 dt \\ &= arg \min_{\mathbf{b}\in\{\mathbf{-1},\mathbf{+1}\}^{\mathbf{K}}} \min_{\substack{A_i>0\\ i=1,\ldots,K}} \mathbf{y^T H^{-1} y + b^T H b - 2 b^T y}; \quad \mathbf{H = ARA} \\ &= sgn\left(arg \min_{x\in R^K} \mathbf{x R^T x - 2 x^T y}\right) = sgn(\mathbf{R^{-1} y})\end{aligned}$$

This essentially reduces to system with single user and but with Gaussian noise $\sigma^2(\mathbf{R^{-1}})_{\mathbf{kk}}$. Hence, the error probability for k^{th} user is given by

$$P_k = Q\left(\frac{A_k}{\sqrt{\sigma^2(\mathbf{R^{-1}})_{\mathbf{kk}}}}\right) \tag{16}$$

A general analysis is carried out for the case when $\mathbf{R}$ is not invertible and a generalized Moore Penrose inverse of $\mathbf{R}$ is used. If $\mathbf{H = ARA}$ and $\mathbf{H^I}, \mathbf{R}^+$ are a generalized Moore Penrose inverses of $\mathbf{H}$ and $\mathbf{R}$ respectively, then the asymptotic efficiency and the near-far resistance is given by:

$$\eta_k^{Dec} = max^2\left(0, \frac{(\mathbf{H^I H})_{\mathbf{kk}} - \sum_{\mathbf{j}\neq\mathbf{k}} |(\mathbf{H^I H})_{\mathbf{kj}}|}{A_k\sqrt{(\mathbf{H^I H H^{I^T}})_{\mathbf{kk}}}}\right) \tag{17}$$

$$\bar{\eta_k}^{Dec} = \frac{1}{R^+_{kk}} \tag{18}$$

Thus the near-far ressitance of Decorrelator detector is same as that of the optimal detector. This can be generalized for the case of asynchronous DS-CDMA where we take the inverse of the general $MK \times MK$ matrix $\mathbf{R}$.

Minimum Mean Square Error (MMSE) Detector

MMSE detector finds the linear operator $\mathbf{L}$ based on the following optimization criterion for the general case of asynchronous CDMA:

$$\mathbf{L} = \mathbf{arg} \min_{\mathbf{L}:\mathbf{R^{MK}}\to\mathbf{R^{MK}}} \mathbf{E}\left[(\mathbf{Ly - b})^{\mathbf{T}}(\mathbf{Ly - b})\right]$$

It is shown that solution to the above optimization problem is given by

$$\mathbf{L} = (\mathbf{RA} + \sigma^{\mathbf{2}}\mathbf{I})^{\mathbf{-1}} \tag{19}$$

where σ^2 is the Gaussian noise power. By rewriting $\mathbf{Ly = LRAb + Ln}$ as $\mathbf{z = Fb + N}$ with $E[\mathbf{N}] = \mathbf{0}$; and $E[\mathbf{NN^T}] = \sigma^{\mathbf{2}}\mathbf{LRL^T}$, an analysis similar to that for conventional receiver can be carried out to obtain all the permormance metric viz: error probability, asymptotic efficiency, and near far resistance.

(B.4) *Comments*:

Linear receivers present a good alternative to the optimal MUD since they reduces complexity by a great extent with only a slight degradatioin in the performance as seen in the simulation results. Complexity per bit is linear in number of users as for detecting block of M bits for K users requires multiplication by as $KM \times KM$ matrix, hence complexity per bit is $\frac{1}{KM}O\left((KM)^2\right) = O(K)$ for fixed block length. However, finding an appropriate matrix $\mathbf{L}$ reqires matrix inversion which has $O(K^3)$ complexity. This is not an issue in an AWGN channel since the matrix inversion needs to be done only once, but if we consider a narrowband fading and try to generalize such scheme, $\mathbf{L}$ will have to be computed once the channel condition changes and hence it will have high complexity.

Deorrelator receiver can be considered to be analogous to a zero forcing equlization and removes MAI completely at the expense of enhancing noise. MMSE detector is alalogous to MMMSE equilizer. At low SNR, it performs slightly better than Decorrelator detector while at high SNR this becomes identical to Decorrelator detector.

A variety of non-linear detectors can be constructed which has a linear detector as its first stages and at higher stages, estimates from the previous stages are used to cancel MAI and give a better estimate (parallel interference cancellation).

Though the linear detectors are low in complexity, for asynchronous case, it is still required to observe the entire block of M bits before processing can begin. Moreover, they require almost the same amount of information as required by the Optimal detector which limits their applicability. In the next section we describe some adaptive scheme which eliminates the need for correct estimates of all the parameters by using a training sequence.

C. Blind Multiuser detectors

Blind adaptive schemes for Multiuser detection are described in [4],[5]. [4] introduces an adaptive MUD which converges (for any initialization) to the MMSE detector and the *only* information required is the desired user's signature waveform and timing which is no more than what is required by the conventional detector. [5] shows that through the use of signal subspace estimation, both the decorrelating filterbank and the linear MMSE filterbank can be obtained blindly, that is they can be estimated from the received signal with the prior knowledge of only the signature waveform of the desired user. We only summarize the approach given in [4].

(B.1) *Assumptions*:
An AWGN channel is considered. It is assumed that signature waveform and timing of the desired user is known.

(B.2) *Main Idea*:
The main idea in [4] is to decompose the impulse response of the linear reciever into the signature waveform of the desired user plus an orthogonal adaptive component. It is shwon than the receiver that results from the minimization of the output energy (MOE) is the MMSE multiuser detector. The convexity of MOE allows global convergence of the iterative algorithms.

(B.3) *Results and Approaches*:
A linear detector for user 1 is characterized by the impulse response $c_1 \in L_2[0,T]$ such that the decision on $\hat{b}_1 = sgn(< r, c_1 >$, where the inner product $< r, c_1 >= \int_0^T r(t)c_1(t)dt$ and $r(t)$ is the received waveform. We write $c_1 = s_1 + x_1$ where $s_1(t)$ is the normalized signature waveform of user 1 and $< s_1, x_1 >= 0$. Set of signals that can be written like this must satisfy $< c_1, s_1 >= ||s_1||^2 = 1$. There is no loss of generality in above notation since we can rule out linear transformations that are orthogonal to the desired signal (they result in error probability equal to 0.5) and the decision scheme is invariant to positive scaling.
The MMSE linear multiuser detector for user 1 is defined as the signal $c_1 \in L_2[0,T]$ that minimizes the mean square error (MSE) $E[(A_1b_1 - < r, c_1 >)^2]$ while the minimum output energy (MOE) linear detector is one which minimizes $E[(< r, c_1 >)^2] = E[(< r, s_1 + x_1 >)^2]$ over all x_1 orthogonal to s_1. Following two observations are the key to the blind implementation.

1) If $MOE(x_1) = E[(< r, s_1 + x_1 >)^2]$ and $MSE(x_1) = E[(A_1b_1 - < r, s_1 + x_1 >)^2]$ then $MSE(x_1) = MOE(x_1) - A_1^2$.
2) $MOE(x_1)$ is strictly convex over set of signals orthogonal to s_1.

The arguments that minimize both $MOE(x_1)$ and $MSE(x_1)$ are the same. This means that it is not necessary to know the data in order to implement a gradient descent algorithm for the minimization of mean-square-error. Moreover, since $MOE(x_1)$ is strictly convex, a global

minimum is reached for any initialization of x_1 orthogonal to s_1. Hence the adaptation rule is given as follows.
Let the observed waveform in the i^{th} interval $[iT, iT+T]$ be denoted by $r[i]$ and the i^{th} output of the conventional receiver be $Z_{MF}[i] =< r[i], s_1 >$ and $Z[i] =< r[i], s_1 + x_1[i-1] >$. Then the gradient of $MOE(x_1) = E[(< r, s_1 + x_1 >)^2]$ is equal to the scaled version of the observation $2 < r, s_1 + x_1 >$ and the component orthognal to s_1 is $r- < y, s_1 > s_1$. Since we need to iterate in the direction of the gradient orthogonal to s_1, the stochastic gradient descent rule is given by

$$x_1[i] = x_i[i-1] - \alpha Z[i](r[i] - Z_{MF}[i]s_1) \tag{20}$$

Moreover, an extension is proposed in [4] for asynchronous case. The case when there is an imperfect knowledge of user's signature is also analyzed.

(B.4) *Comments*:
The receiver proposed in [4] doesn't require training sequence for adaptation. However, the inital convergence time is an issue and needs to be analyzed. Also, in case of fast fading, use of such schem will require that the the value of $x_1[i]$ needs to be updated quickly since the impluse response for the linear receiver depends on channel condition.

IV. Proposed low complexity algorithm and extension of some results

In this section we develope a low complexity algorithm for MUD. Note that one of the two ways that we had suggested for developing a low complexity algorithm was through an application of EM algorithm. However, we found that some related work had already been done on it and we refer readers to [6] for more details on the same approach. We describe below a new local search based algorithm.
*Intution:*Let $\hat{\mathbf{b}}$ be the output of a low complexity MUD algorithm (Decorrelator, MMSE etc) and let $\mathbf{b}^{\mathbf{opt}}$ be the output of Optimal ML-MUD. If our intial hypothesis is reasonable, then $\hat{\mathbf{b}}$ and $\mathbf{b}^{\mathbf{opt}}$ will disagree only at few places. If we can identify the bits which are likely to be in error and apply ML-Detection only for that small set of bits, then it is likely that performance will be close to that of optimal MUD without much increase in complexity.

An obvious way to estimate the bits that are likely to be in error is by computing the log likelihood ratio (LLR). Again assuming independent and equiprobable bits, the application of Bayes rule gives

$$\begin{aligned} l_k &= log\left(\frac{P(b_k=+1|\mathbf{y})}{P(b_k=-1|\mathbf{y})}\right) \\ &= log\left(\frac{P(\mathbf{y}|\mathbf{b_k=+1})\mathbf{P}(\mathbf{b_k=+1})}{P(\mathbf{y}|\mathbf{b_k=-1})\mathbf{P}(\mathbf{b_k=-1})}\right) \\ &= log\left(\frac{P(\mathbf{y}|\mathbf{b_k=+1})}{P(\mathbf{y}|\mathbf{b_k=-1})}\right) \\ &= log\left(\frac{\sum_{\substack{\mathbf{b}\in\{\mathbf{-1,+1}\}^{\mathbf{K}}\\ b_k=+1}} P(\mathbf{y}|\mathbf{b})(\mathbf{2^{1-K}})}{\sum_{\substack{\mathbf{b}\in\{\mathbf{-1,+1}\}^{\mathbf{K}}\\ b_k=-1}} P(\mathbf{y}|\mathbf{b})(\mathbf{2^{1-K}})}\right) \\ &= log\left(\frac{\sum_{\substack{\mathbf{b}\in\{\mathbf{-1,+1}\}^{\mathbf{K}}\\ b_k=+1}} P(\mathbf{y}|\mathbf{b})}{\sum_{\substack{\mathbf{b}\in\{\mathbf{-1,+1}\}^{\mathbf{K}}\\ b_k=-1}} P(\mathbf{y}|\mathbf{b})}\right) \end{aligned}$$

Similarly, likelihood ratio for other bits can be computed. The magnitude of LLR gives an estimate of the confidence associated with the decision, that is, if LLR is a large positive number then bit is mosre likely to be $+1$ and vice versa. Even though its poissible to compute the LLR since $\mathbf{y}|\mathbf{b}$ follows a multivariate normal distribution with mean $\mathbf{RAb}$ and covariance matrix $\sigma^2\mathbf{R}$, it involves computing 2^{K-1} terms and summing them up. Hence, the computational complexity is $O(2^K)$ which is same as that for optimal MUD. However, if we are given $E(b_1|\mathbf{y})$ then following result helps in computing LLR.

$$\begin{aligned} \mu_k = E[b_k|\mathbf{y}] &= (+\mathbf{1})\mathbf{P}(\mathbf{b_k=+1}|\mathbf{y}) + (-\mathbf{1})\mathbf{P}(\mathbf{b_k=-1}|\mathbf{y}) \\ &= \frac{P(\mathbf{y}|\mathbf{b_k=+1})\mathbf{P}(\mathbf{b_k=+1}) - \mathbf{P}(\mathbf{y}|\mathbf{b_k=-1})\mathbf{P}(\mathbf{b_k=-1})}{P(\mathbf{y}|\mathbf{b_k=+1})\mathbf{P}(\mathbf{b_k=+1}) + \mathbf{P}(\mathbf{y}|\mathbf{b_k=-1})\mathbf{P}(\mathbf{b_k=-1})} \\ &= \frac{P(\mathbf{y}|\mathbf{b_k=+1}) - \mathbf{P}(\mathbf{y}|\mathbf{b_k=-1})}{P(\mathbf{y}|\mathbf{b_k=+1}) + \mathbf{P}(\mathbf{y}|\mathbf{b_k=-1})} \end{aligned}$$

This gives

$$l_k = log\left(\frac{1+\mu_k}{1\mu_k}\right) \tag{21}$$

However, using Bayes rule, we have

$$\mu = E[\mathbf{b}|\mathbf{y}] = \frac{\sum_{\mathbf{b}\in\{\mathbf{-1,+1}\}^{\mathbf{K}}} \mathbf{bP}(\mathbf{y}|\mathbf{b})\mathbf{P}(\mathbf{b})}{\sum_{\mathbf{b}\in\{\mathbf{-1,+1}\}^{\mathbf{K}}} \mathbf{P}(\mathbf{y}|\mathbf{b})\mathbf{P}(\mathbf{b})} \tag{22}$$

$$= \frac{\sum_{\mathbf{b}\in\{\mathbf{-1,+1}\}^{\mathbf{K}}} \mathbf{bP}(\mathbf{y}|\mathbf{b})}{\sum_{\mathbf{b}\in\{\mathbf{-1,+1}\}^{\mathbf{K}}} P(\mathbf{y}|\mathbf{b})} \tag{23}$$

Thus, computation complexity of $E[\mathbf{b}|\mathbf{y}]$ is still $O(2^K)$. Thus we try to find an approximate value of μ based on follwong known result.

Minimum mean square error: Suppose x, y are two random variable with joint pdf $P(x, y)$.

Suppose we measure $y = y_{meas}$ and estimate x by $\hat{x} = \phi(y_{meas})$, then the function ϕ which minimizes the mean sqare error $E[||\phi(y) - x||^2]$ is given by $\phi_{mmse} = E(x|y)$. Thus $\hat{x}_{mmse} = E(x|y = meas)$.

For our case, we can treat transmitted bits $\mathbf{b}$ and matched filter output $\mathbf{y}$ as the two random variables and try to estimate $\mathbf{b}$. If we confine $\phi(\mathbf{y}) = \mathbf{Ly},\ \mathbf{L} \in \mathbf{R^K} \times \mathbf{R^K}$, then we get liner MMSE detector. Hence the idea is to use the output of linear MMSE receiver berfore thresholding and use it as an approximation for $E[\mathbf{b}|\mathbf{y}]$. LLR can then be computed from (21). The algorithm is summarized below.

1) Compute the output of the linear MMSE detector $\mathbf{z_{mmse}} = (\mathbf{RA} + \sigma^{\mathbf{2}}\mathbf{I})^{\mathbf{-1}}\mathbf{y}$.
2) Compute the LLR using (21) with $\mu_k = \mathbf{z_{mmse}(k)}$.
3) Identify the top n bits with lowest LLR magnitude. Let S_n denotes the set of such bits and S_{K-n} be the set of remaining $K - n$ bits.
4) New detection rule is: $b_j = sgn(z_j),\ \forall b_j \in S_{K-n}$ and $\mathbf{b} = \mathbf{arg}\ \max_{\forall \mathbf{b_k} \in \mathbf{S_n}} \mathbf{2b^T Ay} - \mathbf{b^T ARAb}$ given $b_j = sgn(z_j),\ \forall b_j \in S_{K-n}$.

The complexity per bit of this scheme is $O(K + 2^n/K)$ since search is performed over n bits only. If the channel state is changing (narrow band fading), then matix inversion will have to be done again when the channel condition changes. However, an improtant observation is that we do not have to find $(RA+\sigma^2 I)^{-1}$ explicitly rather we need the product $(RA+\sigma^2 I)^{-1}y$ which is equivalent to sloving a system of linear equations given by $(RA+\sigma^2 I)x = y$. Hence iterative shcemes such as Gauss-Seidal iteration can be applied. Thus this can be generalized to work in the fading environment also.

V. Simulation Results

Simulations are carried out to compare the performance of various schemes discussed so far. A synchronous CDMA channel with AWGN and 10 users are considered.

First, we consider the case of equal signal power. A performance comparison is done between the conventional detector, decorrelating detector, MMSE detector, the proposed algorithm with (n=3) and the optimal MUD. Results are shown in Figs. 1 and 2 for low and high correlation respectively. In Fig. 1, Gold sequences of length 31 (processing gain) are used as signatures while in Fig. 2 correlation between any pair of users is 0.6 and processing gain is 20. Average of bit error rate (BER) of all users is taken as the performance measure parameter. All the multiuser detection schemes offer substantial improvement over the conventional detector especially in the case of high cross correlation. Moreover, MMSE performs marginally better than the decorrelator at low SNR while the high SNR their performance become nearly same. Moreover, the proposed algorithm performs nearly same as the optimal MUD especially in the case of high cross correlation where curves are nearly indistinguishable.

To study the effect of unequal power, power of user 2 is varied from 0 db to 20 db while keeping the power of all other users fixed at 0 db and SNR fixed at 5 db. Gold sequences of length 31 are used as signatures. Results are shown in Fig. 3. Average of bit error rate (BER) of all users except user 2 is taken as the performance measure parameter. The conventional reveiver is found to perform vary badly in presence of a dominant interferer with agverage error probability reaching 0.5 while the performance of other schemes are almost constant. The proposed algorithm performs much better than a decorrelating and MMSE detector showing that it is has high near far resistance.

VI. Conclusions and future work

A study of the key theoretical foundations of MUD is carried out. A trade off between complexity, performance and assumption about what is know is studied. A new low complexity algorithm for MUD is also described. Simulation results show the performance gains of various shemes.

A more general analysis of the proposed algorithm needs to be carried out to find out the performance in the case of fading environment and asynchronous transmission. The effect of using Gauss Seidal iteration as the first step of proposed algorithm has to be evaluated. An attempt can be made to find out the bound for error probability for the prosposed scheme. Also, a general validaity of the arroximation used for calculating LLR needs to be studied. A further generalization can be in the direction of modifying such schemes for a MIMO system.

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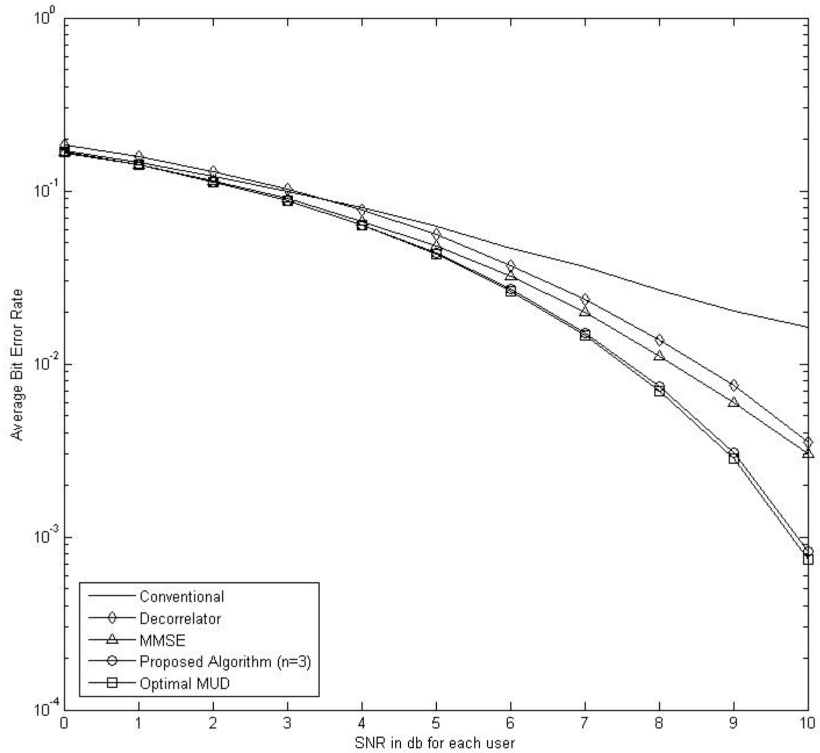

Fig. 1. Comparison of average BER of various schemes for equal signal power and low correlation

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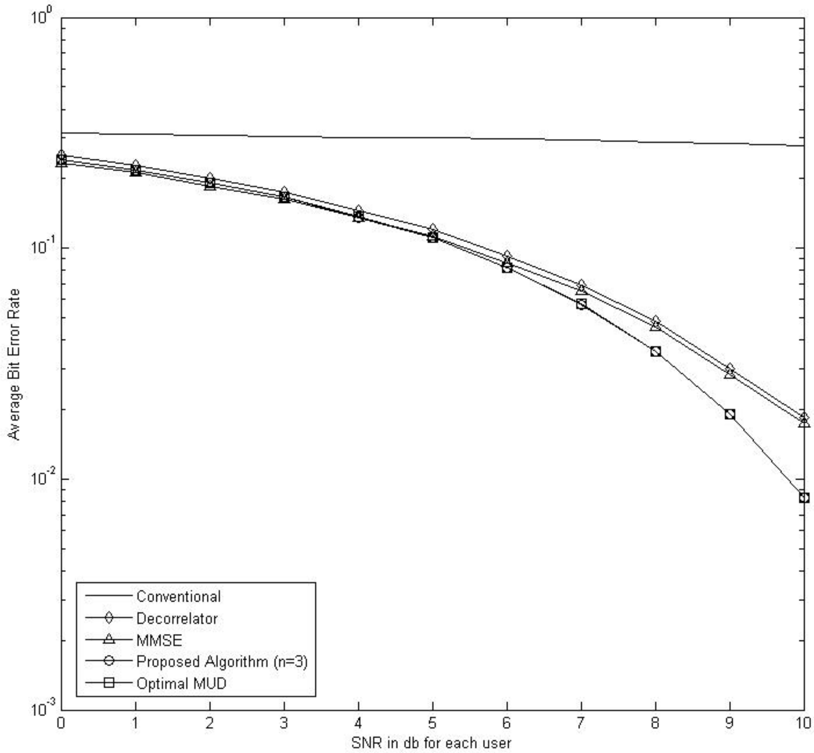

Fig. 2. Comparison of average BER of various schemes for equal signal power and high correlation of 0.6 between any two users

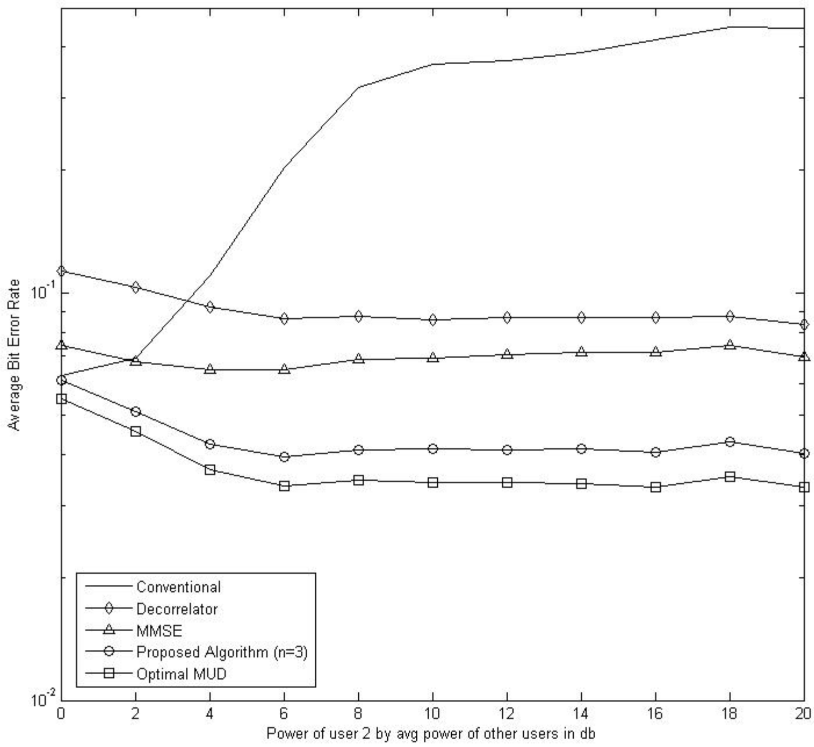

Fig. 3. Performance in the case of varying power of interferers with a fixed SNR of 5db