

Dense Error Correction via ℓ^1 -Minimization

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Abstract

We consider the problem of recovering a non-negative sparse signal $\mathbf{x} \in \mathbb{R}^n$ from highly corrupted linear measurements $\mathbf{y} = A\mathbf{x} + \mathbf{e} \in \mathbb{R}^m$, where \mathbf{e} is an unknown error vector whose nonzero entries may be unbounded. Motivated by the problem of face recognition in computer vision, we will prove that for highly correlated (and possibly overcomplete) dictionaries A , any non-negative, sufficiently sparse signal \mathbf{x} can be recovered by solving an ℓ^1 -minimization problem

$$\min \|\mathbf{x}\|_1 + \|\mathbf{e}\|_1 \quad \text{subject to} \quad \mathbf{y} = A\mathbf{x} + \mathbf{e}.$$

More precisely, if the fraction ρ of errors is bounded away from one and the support of \mathbf{x} grows sublinearly in the dimension m of the observation, then as m goes to infinity, the above ℓ^1 -minimization succeeds for all signals \mathbf{x} and almost all sign-and-support patterns of \mathbf{e} . This result suggests that accurate recovery of sparse signals is possible and computationally feasible even with nearly 100% of the observations corrupted. The proof relies on a careful characterization of the faces of a convex polytope spanned together by the standard cross polytope and a set of iid Gaussian vectors with nonzero mean and small variance, which we call the “cross-and-bouquet” model. Simulations and experimental results corroborate our findings, and suggest extensions to our result.

This is joint work with Yi Ma (UIUC).

Speaker Bio: John Wright is a PhD candidate in the Department of Electrical and Computer Engineering at the University of Illinois at Urbana-Champaign. His research interests include sparse representation and compressed sensing, data segmentation, lossy minimum description length methods, and their applications in computer vision, including face recognition, image and motion segmentation, and image reconstruction. He has interned at Microsoft Research in Asia, and Microsoft Live Labs in Redmond, and is currently a Microsoft Live Labs fellow.